

Theoretical Examination of the European Union: A General Equilibrium Simulation

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Keywords: *Defense, Nation, General equilibrium, Lindahl mechanism, Simulation*

Introduction

In 2012, the European Union, EU, was awarded the Nobel Prize for Peace. With the deep remorse of the World Wars I and II, the European countries have attempted to remove the walls of countries, with the final purpose of integrating themselves into one society. So far, economic integration appears to have been advanced, as represented by the introduction of Euro. The political integration, such as unification of armed forces may be attempted in later stage.

Fukiharu [2011] examined the strategy of European countries, utilizing general equilibrium theory. He constructed a primitive two-region framework, in which two regions commonly face the destruction by intruders. Each region has civilian and military goods, providing governmental services including regional defense as a public good, while the members, consisting of households and firms, pay the provision cost of public good in terms of Lindahl mechanism (taxation). In examining the process to the formation of a nation, he considered 2 types of the process. The first type first adopts the economic integration: the construction of national market of the consumption good, later adopting the political integration: the construction of national armed force. The second type first adopts the political integration, later adopting economic integration. Utilizing a simulation approach in terms of general equilibrium theory, he showed that in the first type the transition from the isolated regions to the economic integration is Pareto improving, and the transition further to the political integration is also Pareto improving. Meanwhile, he showed that in the second type the transition from the isolated regions to the political integration is Pareto improving, and the transition further to the economic integration is *not* Pareto improving. Utilizing this result, he asserted that the

strategy introduced by EU is reasonable.

The aim of this paper is to extend the two-region framework to the three-region framework, since as the model structure becomes sophisticated, the assertion sometimes is not guaranteed. This phenomenon is well-known for economists. For instance, in the two-commodity case, the global stability for the market prices is guaranteed, while in the three-commodity case the assertion is not guaranteed (Arrow and Hahn [1971]). In this extension, the amendment of insufficient analysis in Fukiharu [2011] is also attempted.

In this paper, in Section 1, the isolated three-region general equilibrium model with Lindahl mechanism is constructed. In Section 2, the economic integration of commodity good without political integration is examined in the same economic model. In Section 3, the political integration without economic integration is examined in the same economic model. Finally, in Section 4, the case of economic integration and political integration is examined in the same economic model. Comparing the utility variation of the society members, it is concluded if the assertion in the two-region framework is guaranteed in the three-region framework.

In the present paper, the computation of simulation for the two-region case is done in Fukiharu [2014a], and the one for the three-region case is done in Fukiharu [2014b].

1. Regions under Isolated Defense

When the two neighboring regions, A and B are hostile to each other, the existence of armed force in each region implies the “external diseconomy” to each other. When one region’s, say Region A’s, armed force increases it reduces Region B’s output, although it raises the utility level of the Region A’s household and vice versa. This aspect was featured in Fukiharu [2005]. In the present paper, there are three neighboring regions, A, B, and C, assumed to face the same hostile intruder. In this case, the raised armed force in each region raises its own output, by offsetting the intruder’s assault on their territories, while raising the utility of their own households. Thus, the armed force as the “public good” is featured in this paper. When the “public good” aspect of armed force is considered as in this paper, its optimal level may be attained by Lindahl mechanism. We start with the analysis in Region A.

Region A

Population in Region A, L_{0A} , is assumed to be 100. Region A faces the hostile intruder X. Region A must offset the effect of attack by intruder X. Without the counterattack by Region A, the intruder X invades freely into the Region A, destroying the production facilities there. With the increase of the armed force in Region A, the invasion by the intruder X could be reduced, thus, raising output for

Region A. There are two industries in Region A. The industry 1 is the civilian good industry, which is owned by the households, producing the civilian good, x_{cA} , hiring labor, l_{cA} , where output depends on the level of armed force, d_A . Production function, $f_{1A} [l_{cA}, d_A]$, is assumed to be of the following Cobb-Douglas type.

$$x_{cA} = f_{1A} [l_{cA}, d_A] = l_{cA}^{\alpha_{1A}} d_A^{\alpha_{2A}} \quad \alpha_{1A} + \alpha_{2A} < 1 \quad (1)$$

The industry 2 is the military good industry, which is owned by the Region A's government, producing the military good, m_A , utilizing civilian good, x_{mA} , and labor, l_{mA} . Production function, $f_{2A} [x_{mA}, l_{mA}]$, is assumed to be of the following Cobb-Douglas type.

$$m_A = f_{2A} [x_{mA}, l_{mA}] = x_{mA}^{\beta_1} l_{mA}^{\beta_2} \quad \beta_1 + \beta_2 \leq 1 \quad (2)$$

It is assumed that f_{2A} does not depend on d_A .

Region A's level of armed force, d_A , consists of military good, m_A , and military personnel, v_A . This relation is defined by the following defense function.

$$d_A = f_{3A} [m_A, v_A]$$

The government provides the level of armed force, d_A^0 , by the minimum cost principle subject to $d_A^0 = f_{3A} [m_A, v_A]$ where the price of civilian good is p_{cA} and the wage rate is w , on the assumption that the military personnel are provided with the civilian wage rate. Thus, given d_A^0 , the government computes the demand for civilian good, x_{mA}^D , the demand for labor, l_{mA}^D , and the demand for military personnel, v_A^D , given p_{cA} and w , by solving

$$\min p_{cA} x_{mA} + w(l_{mA} + v_A) \quad \text{subject to (2) and } d_A^0 = f_{3A} [m_A, v_A].$$

In this section, suppose that the defense function is specified by the CES (Constant Elasticity of Substitution) type:

$$f_{3A} [m_A, v_A] = (m_A^{-\tau} + v_A^{-\tau})^{-n/\tau}, \quad n=1, \tau = -1/2. \quad (3)$$

Furthermore, other parameters are stipulated by

$$\alpha_{1A} = \alpha_{2A} = 1/3, \quad \beta_1 = \beta_2 = 1/2. \quad (4)$$

All Volunteer Armed Force System in Region A

Military industry in Region A

Military Industry is assumed to employ the military personnel at the wage rate w . The demand functions, $\{x_{mA}^D, l_{mA}^D, v_A^D\}$, are analytically computed by a simple Lagrangian method with p_{cA} , w , and d_A^0 parameters. Utilizing these demand functions, the minimum cost function for providing the region A's level of armed force, d_A^0 , $cd[d_A^0]$, is analytically derived with p_{cA} and w parameters.

Civilian Industry in Region A

Next, we examine the public good aspect of armed force. The service of armed force raises output of consumption good by offsetting the damage from invasion. It also raises the utility level of household. Thus, it has the property of public good. Lindahl mechanism has been known to achieve the optimum provision of public good. In this mechanism, the government announces arbitrary shares of burden for providing public good to each member of the society. Each member replies with the desired level of armed force. There is no guarantee that those replied levels of armed force are the same. If they are not the same, the government announces different shares of burden to each member. Each member, then, replies with the desired level of armed force. If they are not the same again, the government announces different shares of burden to each member. Continuing this process, the government searches for the consensus of the level of armed force among the members. The consumption good industry also shares the burden of keeping the armed force. Suppose that t_{fA} is the share of burden for the consumption good industry. The behavior of the industry is the following profit maximization.

$$\text{Max } \pi_{cA} = p_{cA} x_{cA} - w l_{cA} - t_{fA} cd[d_A].$$

From this maximization, demand function for labor, l_{cA}^D , the demand function for the armed force, d_{fA}^D , and supply function, x_{cA}^S , are analytically derived with p_{cA} , w , and t_{fA} parameters. The resulting (expected) maximum profit, π_{cA} , is computed with p_{cA} , w , and t_{fA} parameters. This profit is distributed to the household in Region A.

Aggregate Household in Region A

The last agent is the (aggregate) household. As in the traditional approach, it maximizes utility subject to income constraint. Utility function is assumed to be of the following Cobb-Douglas type.

$$u = U_A[x_{cA}, d_A] = x_{cA}^{\gamma_{1A}} d_A^{\gamma_{2A}} \quad \gamma_{1A} + \gamma_{2A} = 1.$$

The government hears the desired level of armed force from the household, by presenting the share of burden for the household as $t_{hA}=1-t_{fA}$. The household's behavior is formulated as in what follows.

$$\text{Max } U_A[x_{cA}, d_A] \quad \text{s.t. } p_{cA} x_{cA} + (1-t_{fA})cd[d_A]=wL_{0A}+\pi_{cA}$$

In this paper, the household has no choice between working at civilian industry or military industry, enjoying leisure hours, and volunteering for the military personnel. In a sense, the fixed number of enjoyable leisure hours is subtracted from the initial holding of leisure hours. The numbers (or hours) of volunteers required for the armed force and required workers at the military industry are computed by the government and a part of the household must serve in the armed force or work at the military industry, with the military wage paid at the civilian wage rate, w . Thus, given p_{cA} , w and t_{fA} , the household expresses its desired armed force, proceeding to the government's role in which government computes the number (hours) of required military personnel and the one of workers at the military industry by the minimum cost principle, and a part of the household must serve in the armed force or work at the military industry. The remaining workers (or working hours) are employed at the civilian industry.

For the purpose of simulation, in this section, parameters are stipulated.

$$\gamma_{1A}=\gamma_{2A}=1/2. \quad (5)$$

From this maximization, the demand function for consumption good; x_{chA}^D , and the demand function for the armed force; d_{hA}^D , are derived with p_{cA} , w , and t_{fA} parameters.

General Equilibrium with Lindahl-Walras Mechanism in Region A

So far we derived demand and supply functions with p_{cA} , w , and t_{fA} parameters. Since the service of defense is a public good, the Region A's government cannot provide the service if d_{hA}^D and d_{fA}^D are different from each other. Lindahl [1919] asserted that the equality of d_{hA}^D and d_{fA}^D is possible by the proper selection of t_{fA} . Thus, the Lindahl mechanism selects $0 \leq t_f \leq 1$, which satisfies

$$d_{hA}^D = d_{fA}^D = d_A.$$

The selection of t_{fA} , however, must be done jointly with p_{cA} and w , since in order to compute the minimum cost for providing d_A , prices, p_{cA} and w , must be known beforehand. Thus, p_{cA} , w , and t_{fA} as well as d_A must be determined in the context of general equilibrium. This extended Lindahl mechanism is named the Lindahl-Walras mechanism, or L-W mechanism, in this paper.

The consumption good market is in equilibrium if the following equation holds.

$$x_{cA}^S = x_{mA}^D + x_{chA}^D .$$

Labor market is in equilibrium if the following equation holds.

$$L_{0A} = l_{mA}^D + v_A^D + l_{cA}^D .$$

In the computation of general equilibrium with Lindahl mechanism, $\{p_{cA}^*, t_{fA}^*, d_A^*\}$, the Newton method is utilized with the normalization of $w=1$. The Newton method computes $\{p_{cA}^*, t_{fA}^*, d_A^*\}$ as in what follows.

$$p_{cA}^* = 5.18048, t_{fA}^* = -0.272514, d_A^* = 103.148 \quad (6)$$

Alternatively, $\{p_{cA}^*, t_{fA}^*, d_A^*\}$, the solution for the L-W mechanism, can be computed by the following differential equations, where s is time.

$$\begin{aligned} dp_{cA}[s]/ds &= x_{mA}^D + x_{chA}^D - x_{cA}^S \\ dt_{fA}[s]/ds &= d_{fA}^D - d_{fA}^D \\ dd_A[s]/ds &= L_{0A} - (l_{mA}^D + v_A^D + l_{cA}^D) \end{aligned} \quad (7)$$

The solution in (6) is derived by the Newton method with the proper selection of initial values. This selection is not easy when the number of variables becomes large. This difficulty can be reduced if we compute the trajectories of variables on the differential equations in (7) starting from arbitrary initial values, selecting the values of the trajectories when s is large as the initial values for the Newton method.

The eigenvalues of the Jacobian matrix of (7) at (6) are, -830.233 , -3.96109 , and -0.790576 . Thus, (5) is locally stable. Indeed, the trajectory of $d_A[s]$ on (7) is depicted in Fig.1.

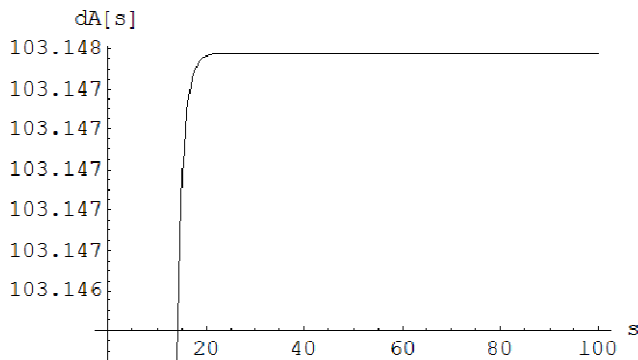


Fig.1: Trajectory of $d_A[s]$

The “all volunteer army” utility level of the (aggregate) household at (5), u_A^* , is computed as in what follows.

$$u_A^* = 34.9997. \quad (8)$$

Region B

Population in Region B, L_{0B} , is assumed to be 200. Region B also faces the hostile intruder X. Region B must offset the effect of attack by intruder X. Without the counterattack by Region B, the intruder X invades freely into the Region B. With the increase of the armed force in Region B, the invasion by the intruder X could be reduced, thus, raising output for Region B. There are two industries in Region B. The industry 1 is the civilian good industry, which is owned by the households, producing the civilian good, x_{cB} , hiring labor, l_{cB} , where output depends on the level of armed force, d_B . Production function, $x_{cB} = f_{1B} [l_{cB}, d_B] = l_{cB}^{\alpha_{1B}} d_B^{\alpha_{2B}}$, is assumed to be of the Cobb-Douglas type. In this section, the same parameters for production functions are specified as in Region A: i.e. $\alpha_{1B} = \alpha_{2B} = 1/3$. The industry 2 is the military good industry, which is owned by the Region B's government, producing the military good, m_B utilizing civilian good, x_{mc1B} , and labor, l_{mB} . Production function, $m_B = f_{2B} [x_{mB}, l_{mB}] = x_{mB}^{\beta_1} l_{mB}^{\beta_2}$, is assumed to be of the Cobb-Douglas type with the same parameter as in Region A: i.e. $\beta_1 = \beta_2 = 1/2$. It is assumed that $m_B = f_{2B} [x_{mB}, l_{mB}]$ does not depend on d_B . Region B's level of armed force (or “defense”), d_B , consists of military good, m_B , and military personnel, v_B , with definition, $d_B = f_{3B} [m_B, v_B]$, is assumed to be of the CES type, whose functional form is exactly the same as in Region A: i.e. $f_{3B} [m_B, v_B] = (m_B^{-\tau} + v_B^{-\tau})^{-n/\tau}$, $n=1$, $\tau = -1/2$. The government provides the level of the armed force by the minimum cost principle in exactly the same as in Region A, with A replaced by B where the price of civilian good is p_{cB} and the wage rate is w , on the assumption that military personnel are provided with the civilian wage rate. Thus, given d_B^0 , the government computes the demand for civilian good, x_{mB}^D , the demand for labor, l_{mB}^D , and the demand for military personnel, v_B^D , given p_{cB} and w , in exactly the same way as in Region A with A replaced by B. From the profit maximization, civilian industry (industry 1)'s demand function for labor, l_{cB}^D , its demand function for the armed force, d_{fB}^D , and its supply function, x_{cB}^S , are derived with p_{cB} , w , and t_{fB} parameters. (Aggregate) household in Region B is assumed to have the same utility function as in Region A: i.e. $u = U_B [x_{cB}, d_B] = x_{cB}^{\gamma_{1B}} d_B^{\gamma_{2B}}$ with $\gamma_{1B} = \gamma_{2B} = 1/2$. From the utility maximization subject to income constraint, household's demand function for consumption good, x_{chB}^D , and the demand function for the armed force, d_{hB}^D are derived with p_{cB} , w , and t_{fB} parameters.

In Region B, under the L-W mechanism, the general equilibrium, $\{p_{cB}^*, t_{fB}^*, d_B^*\}$ are derived by

the Newton method as in what follows.

$$p_{cB}^*=6.52594, t_{fB}^*=0.270461, d_B^*=201.196 \quad (9)$$

Alternatively, $\{p_{cB}^*, t_{fB}^*, d_B^*\}$ can be computed by the differential equations. The eigenvalues of the Jacobian matrix of differential equations at (13) are, -1628.61 , -4.95056 , and -0.806864 . Thus, the L-W mechanism in terms of differential equations is locally stable.

The “all volunteer army” utility level of the (aggregate) household at (9), u_B^* , is computed as in what follows.

$$u_B^*= 61.5186. \quad (10)$$

Region C

Population in Region C, L_{0C} , is assumed to be 300. Region C also faces the hostile intruder X. In this section, exactly the same assumptions are made for Region C. Thus, exactly the same explanation applies with B replaced by C: i.e. $x_{cC} = f_{1C} [l_{cC}, d_C] = l_{cC}^{\alpha_{1C}} d_C^{\alpha_{2C}}$ with $\alpha_{1C}=\alpha_{2C}=1/3$, $m_C = f_{2C} [x_{mC}, l_{mC}] = x_{mC}^{\beta_1} l_{mC}^{\beta_2}$ with $\beta_1=\beta_2=1/2$, $d_C = f_{3C} [m_C, v_C] = (m_C^{-\tau} + v_C^{-\tau})^{-n/\tau}$ with $n=1$, $\tau=-1/2$, and $u = U_C[x_{cC}, d_C] = x_{cC}^{\gamma_{1C}} d_C^{\gamma_{2C}}$ with $\gamma_{1C}=\gamma_{2C}=1/2$.

In Region C, under the L-W mechanism, the general equilibrium, $\{p_{cB}^*, t_{fB}^*, d_B^*\}$ are derived by the Newton method as in what follows.

$$p_{cC}^*=7.46917, t_{fC}^*=0.269332, d_C^*=297.721 \quad (11)$$

Alternatively, $\{p_{cB}^*, t_{fB}^*, d_B^*\}$ can be computed by the differential equations. The eigenvalues of the Jacobian matrix of differential equations at (11) are, -2417.87 , -5.63905 , and -0.816166 . Thus, the L-W mechanism in terms of differential equations is locally stable.

The “all volunteer army” utility level of the (aggregate) household at (11), u_C^* , is computed as in what follows.

$$u_C^*= 85.6152. \quad (12)$$

2. Defense Integration (without Market Integration) of Regions A, B, and C

In this section, the defense integration among the three regions is examined, where the civilian

industries operate in each region, with outputs consumed only in their own regions, so that there is no national market for the civilian good. It is assumed, however, that labors freely migrate among the regions. There is population of $L_{0A} + L_{0B} + L_{0C}$ in this integration. Production functions of the civilian industries are the same between the three regions. Production function of the military industry is also the same between the three regions. It is assumed that f_2 does not depend on d as before. In this integration, the level of armed force, d , is a function of military goods and personnel as stipulated in (3).

The defense integration in this section does not imply the formation of a nation in the sense that civilian goods produced by each region are consumed only in each region, so that the civilian good's price may be different across the regions. Let p_{cA} be the price of civilian goods in region A, while p_{cB} and p_{cC} are the prices of civilian goods in Regions B and C, respectively. Since $L_{0A} < L_{0B} < L_{0C}$ is assumed, $p_{cA} < p_{cB} < p_{cC}$ may well happen. Indeed, when there is no defense integration among the three regions, the price in Region A is the smallest, as shown in the previous section. In what follows, the integrated military industry is assumed to believe that the price inequality holds.

Military Industry in the Defense Integration

The allied government provides the level of the armed force by the minimum cost principle where the price of civilian good is p_{cA} and the wage rate is w , on the assumption that the military personnel can be freely recruited from Regions A through C with the civilian wage rate. Thus, given d^0 , the government computes the demand for civilian good, x_m^D , the demand for labor, l_m^D , and the demand for military personnel, v^D , with p_{cA} and w parameters, by the cost minimization. In this section, parameters are specified by (4). Utilizing these demand functions, the minimum cost function for providing the coalition's level of armed force, d^0 , $cd[d^0]$, is derived, with p_{cA} and w parameters.

Civilian Industry in Region A under the Defense Integration

The consumption good industry in Region A also shares the burden of keeping the armed force. Suppose that t_{fA} is the share of burden for the consumption good industry in Region A. The behavior of the industry is the following profit maximization.

$$\text{Max } \pi_{cA} = p_{cA} x_{cA} - w l_{cA} - t_{fA} cd[d_{fA}].$$

From this maximization, demand function for labor, l_{cA}^D , the demand function for the armed force, d_{fA}^D , and supply function, x_{cA}^S , are analytically derived with p_{cA} , w , and t_{fA} parameters. The resulting (expected) maximum profit, π_{cA} , is computed with p_{cA} , w , and t_{fA} parameters. This profit is distributed to the household in region A.

Civilian Industry in Region B and C under the Defense Integration

The consumption good industry in Region B also shares the burden of keeping the armed force. Suppose that t_{fB} is the share of burden for the consumption good industry in Region B. The behavior of the industry is the same profit maximization as in the civilian industry in Region A. From this maximization, demand function for labor, l_{cB}^D , the demand function for the armed force, d_{fB}^D , and supply function, x_{cB}^S , are analytically derived with p_{cB} , w , and t_{fB} parameters. The resulting (expected) maximum profit, π_{cB} , is computed with p_{cB} , w , and t_{fB} parameters. This profit is distributed to the household in Region B.

In exactly the same way, in Region C, civilian industry's demand function for labor, l_{cC}^D , the demand function for the armed force, d_{fC}^D , and supply function, x_{cC}^S , are analytically derived with p_{cC} , w , and t_{fC} parameters. The resulting (expected) maximum profit, π_{cC} , is computed with p_{cC} , w , and t_{fC} parameters.

Household in Region A under the Defense Integration

The (aggregate) household in Region A maximizes utility subject to income constraint. Utility function is assumed to be the same as in the preceding sections. The government asks the desired level of armed force from the household, by presenting the share of burden for the household as t_{hA} . The household's behavior is formulated as in what follows.

$$\text{Max } U_A[x_{cA}, d_A] \quad \text{s.t. } p_{cA} x_{cA} + t_{hA} c d[d_A] = w L_{0A} + \pi_{cA}$$

Thus, given p_{cA} , w and t_{hA} , the household expresses its desired armed force, on which the government computes the number (hours) of required military personnel and the one of workers at the military industry by the minimum cost principle, and a part of the household must serve in the armed force or work at the military industry. The remaining workers (or working hours) are employed at the civilian industry.

For the purpose of simulation, in this paper, parameters are stipulated by (5). From this maximization, demand function for consumption good, x_{chA}^D , and the demand function for the armed force, d_{hA}^D , is derived.

Household in Region B and C under the Defense Integration

The (aggregate) household in Region B maximizes utility subject to income constraint. Utility function is assumed to be the same as in the previous sections. The government asks the desired level of armed force from the household, by presenting the share of burden for the household as t_{hB} . Thus, given p_{cB} , w , and t_{hB} , the household expresses its desired armed force. From this maximization, demand function for consumption good, x_{cB}^D , and the demand function for the armed force, d_B^D , is

derived.

In exactly the same way, given p_{cC} , w , and t_{hC} , the household in Region C expresses its desired armed force. From this maximization, demand function for consumption good, x_{cC}^D , and the demand function for the armed force, d_C^D , is derived.

General Equilibrium with L-W Mechanism under the Defense Integration

So far, we have derived demand and supply functions with p_{cA} , p_{cB} , p_{cC} , w , t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , as the parameters. In L-W mechanism, the coalition government selects $0 \leq t_{fA} \leq 1$, $0 \leq t_{fB} \leq 1$, $0 \leq t_{fC} \leq 1$, $0 \leq t_{hA} \leq 1$, $0 \leq t_{hB} \leq 1$, $0 \leq t_{hC} \leq 1$, $t_{fA} + t_{fB} + t_{fC} + t_{hA} + t_{hB} + t_{hC} = 1$, which guarantees

$$d^0 = d_{fA}^D = d_{fB}^D = d_{fC}^D = d_{hA}^D = d_{hB}^D = d_{hC}^D.$$

The selection of t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , however, must be done jointly with p_{cA} , p_{cB} , p_{cC} , w , since in order to compute the minimum cost for providing d^0 , prices; p_{cA} and w , must be known beforehand. Thus, p_{cA} , p_{cB} , p_{cC} , w , t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , as well as d^0 must be determined in the context of general equilibrium. The consumption good market is in equilibrium if the following equation holds.

$$x_{cA}^S = x_{mA}^D + x_{chA}^D. \quad (\text{Region A})$$

$$x_{cB}^S = x_{chB}^D. \quad (\text{Region B})$$

$$x_{cC}^S = x_{chC}^D. \quad (\text{Region C})$$

Labor market is in equilibrium if the following equation holds.

$$L_{0A} + L_{0B} + L_{0C} = l_m^D + v^D + l_{cA}^D + l_{cB}^D + l_{cC}^D$$

In the computation of general equilibrium with L-W mechanism, the Newton method is utilized with the normalization of $w=1$. This solution; $\{p_{cA}^{*C}, p_{cB}^{*C}, p_{cC}^{*C}, t_{fA}^{*C}, t_{fB}^{*C}, t_{fC}^{*C}, t_{hA}^{*C}, t_{hB}^{*C}, t_{hC}^{*C}, d^{0*C}\}$ is computed as follows.

$$p_{cA}^{*C}=4.09439, \quad p_{cB}^{*C}=4.0792, \quad p_{cC}^{*C}=5.34527, \quad t_{fA}^{*C}=0.078818, \quad t_{fB}^{*C}=0.0783798, \\ t_{fC}^{*C}=0.11757, \quad t_{hA}^{*C}=0.137384, \quad t_{hB}^{*C}=0.235139, \quad t_{hC}^{*C}=0.352709, \quad d^{0*C}=636.44 \quad (13)$$

Alternatively, the solution, (13), can be derived by the following differential equations.

$$dp_{cA}[s]/ds = x_{mA}^D + x_{chA}^D - x_{cA}^S$$

$$\begin{aligned}
dp_{cB}[s]/ds &= x_{cB}^D - x_{cB}^S \\
dp_{cC}[s]/ds &= x_{cC}^D - x_{cC}^S \\
dt_{fA}[s]/ds &= d_{fA}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dt_{fB}[s]/ds &= d_{fB}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dt_{fC}[s]/ds &= d_{fC}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dt_{hA}[s]/ds &= d_{hA}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dt_{hB}[s]/ds &= d_{hB}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dt_{hC}[s]/ds &= d_{hC}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\
dd^0[s]/ds &= L_{0A} + L_{0B} + L_{0C} - (l_{mA}^D + v_A^D + l_{cA}^D + l_{cB}^D + l_{cC})
\end{aligned} \tag{14}$$

The eigenvalues of the Jacobian matrix of (14) at (13) are

$$\begin{aligned}
&-16206.1, -13108.9, -7604.52, -3787.72, -2187.38, -10.7459, -9.51876, -8.28577, \\
&-0.78686, 0
\end{aligned}$$

The trajectory of $d^0[s]$ on (14) converge to (13) as shown by Fig. 2.

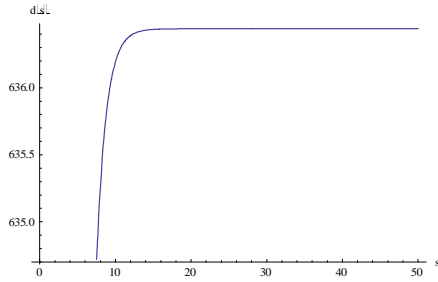


Fig.2: Trajectory of $d^0[s]$

For the comparison with isolated defense case, the regions' utility levels in this integration, u_A^{*C} , u_B^{*C} , and u_C^{*C} , are computed as in what follows.

$$u_A^{*C} = 104.395, u_B^{*C} = 136.83, u_C^{*C} = 146.396 \tag{15}$$

The comparison between (8), (10), (12), and (15) shows that by this political integration each region achieves higher utility level than that for isolated defense case. Thus, this integration is Pareto-improving, which is a feasibility condition of coalition. It is examined next if the resource allocation in this alliance is Pareto-optimal. If it is Pareto optimal, $u_A^{*C} = 104.395$ must be the maximal utility level for Region A, given the utility levels for Region B at $u_B^{*C} = 136.83$ and for

Region C at $u_C^{*C} = 146.396$. Thus, $u_A^{*C} = 104.395$ must be the solution to the following maximization.

$$\begin{aligned} & \text{Max } U_A[x_{cA}, d] \\ & \text{s.t. } U_B[x_{cB}, d] = u_B^{*C}, U_C[x_{cC}, d] = u_C^{*C}, d = f_{3A}[m, v], m = f_{2A}[x_m, l_m], x_m + x_{cA} + x_{cB} + \\ & x_{cC} = f_{1A}[l_{cA}, d] + f_{1B}[l_{cB}, d] + f_{1C}[l_{cC}, d], l_{cA} + l_{cB} + l_{cC} + l_m + v = L_{0A} + L_{0B} + L_{0C} \end{aligned}$$

By the classical Lagrangian method, the maximal solution for can be computed as in what follows.

$$\begin{aligned} x_{cA}^{CO} = 27.6373, d^{CO} = 591.634, x_{cB}^{CO} = 31.6454, x_{cC}^{CO} = 36.2249, v^{CO} = 408.517, x_m^{CO} = 6.8785, \\ l_m^{CO} = 41.5526, l_{cA}^{CO} = 69.5027, l_{cB}^{CO} = 53.5646, l_{cC}^{CO} = 4.68106. \end{aligned} \quad (16)$$

The utility level under (16), u_A^{CO} , is computed as in what follows.

$$u_A^{CO} = 127.872 > u_A^{*C} = 104.395$$

Thus, the defense integration is *not* Pareto-optimal. The reason for the non-Pareto optimality appears to stem from the isolated markets for the civilian industries. If this separation is overcome, we may well achieve the Pareto optimality. Before proceeding to this examination, we must examine another type of integration.

3. Market Integration (without Defense Integration) of Regions A, B, and C

In this section, market integration (without defense integration) among the three regions is examined, where the civilian industries operate in each region, while civilian goods produced in each region are consumed in the integrated (national) market, so that there is a market for the civilian good. In Fukiharu [2011] it was assumed that labors do not migrate between the regions. Modifying this assumption, it is assumed in this section that labors freely migrate between the regions. Production functions of the civilian industries are the same as in the previous sections. The defense, however, is provided separately for each region. Production function of the military industry is stipulated in (3). It is assumed that f_2 does not depend on d as before. Each region constructs the regional defense by the L-W mechanism.

Each government provides the level of the armed force by the minimum cost principle where the price of civilian good is p_c and the wage rate is w , on the assumption that the military personnel can be freely recruited from three regions with the civilian wage rate. Under the minimum cost principle,

given d_A , the government in Region A computes the demand for civilian good, x_{mAA}^D , the demand for labor, l_{mAA}^D , and the demand for military personnel, v_{AA}^D , given p_c and w . In this section, parameters are stipulated by (5). Utilizing these demand functions, the minimum cost function for providing the Region A's level of armed force, d_A , $cd[d_A]$, is derived.

In the same way, given d_B , the government in Region B computes the demand for civilian good, x_{mBB}^D , the demand for labor, l_{mBB}^D , and the demand for military personnel, v_{BB}^D , given p_c and w , by the cost minimization. Utilizing these demand functions, the minimum cost function for providing the Region B's level of armed force, d_B , $cd[d_B]$, is derived.

In exactly the same way, given d_C , the government in Region C computes the demand for civilian good, x_{mCC}^D , the demand for labor, l_{mCC}^D , and the demand for military personnel, v_{CC}^D , given p_c and w , by the cost minimization. Utilizing these demand functions, the minimum cost function for providing the Region C's level of armed force, d_C , $cd[d_C]$, is derived.

The consumption good industry in Region A shares the burden of keeping the armed force along with the household in Region A. Suppose that t_{fAA} is the share of burden for the consumption good industry in Region A, while $1-t_{fAA}$ is the share of burden for the household in Region A. The behavior of the industry is the following profit maximization.

$$\text{Max } \pi_{cA} = p_c x_{cA} - w l_{cA} - t_{fAA} cd[d_{fA}]$$

From this maximization, demand function for labor, l_{cAA}^D , the demand function for the armed force, d_{fAA}^D , and supply function, x_{cAA}^S , are derived.

The resulting (expected) maximum profit, π_{cA} , is distributed to the household in Region A.

The consumption good industry in Region B also shares the burden of keeping the armed force along with the household in Region B. Suppose that t_{fBB} is the share of burden for the consumption good industry in Region B, while $1-t_{fBB}$ is the share of burden for the household in Region B. The behavior of the industry is the same profit maximization. From this maximization, demand function for labor, l_{cBB}^D , the demand function for the armed force, d_{fBB}^D , and supply function, x_{cBB}^S , are derived.

In exactly the same way, the consumption good industry in Region C also shares the burden of keeping the armed force along with the household in Region C. Suppose that t_{fCC} is the share of burden for the consumption good industry in Region C, while $1-t_{fCC}$ is the share of burden for the household in Region C. The behavior of the industry is the same profit maximization. From this maximization, demand function for labor, l_{cCC}^D , the demand function for the armed force, d_{fCC}^D , and supply function, x_{cCC}^S , are derived. The resulting (expected) maximum profit, π_{cC} , is distributed to the household in region C.

The (aggregate) household in Region A maximizes utility subject to income constraint. Utility

function is assumed to be the same as in the previous sections. The government of Region A asks household about the desired level of armed force, by presenting the share of burden for the household as $t_{hAA}=1-t_{fAA}$. The household's behavior is formulated as in what follows.

$$\max U_A[x_{cA}, d_A] \quad \text{s.t.} \quad p_c x_{cA} + t_{hAA}cd[d_A] = wL_{0A} + \pi_{cA}$$

For the purpose of simulation, in this paper, parameters are stipulated by (5). Thus, given p_c , w and t_{hAA} , demand function for consumption good, x_{chAA}^D , and the demand function for the armed force, d_{hAA}^D , are derived.

The (aggregate) household in Region B maximizes utility subject to income constraint. Utility function is assumed to be the same as in the preceding sections. The government of Region B asks household about the desired level of armed force, by presenting the share of burden for the household as $t_{hBB}=1-t_{fBB}$. The household's behavior is the same utility maximization under the income constraint: i.e. given p_c , w and t_{hBB} , demand function for consumption good, x_{chBB}^D , and the demand function for the armed force, d_{hBB}^D , are derived.

In exactly the same way, the (aggregate) household in Region C maximizes utility subject to income constraint. Utility function is assumed to be the same as in the preceding sections. The government of Region C asks household about the desired level of armed force, by presenting the share of burden for the household as $t_{hCC}=1-t_{fCC}$. The household's behavior is the same utility maximization under the income constraint: i.e. given p_c , w and t_{hCC} , demand function for consumption good, x_{chCC}^D , and the demand function for the armed force, d_{hCC}^D , are derived.

In the above examination, we derived demand and supply functions with p_c , w , t_{fAA} , t_{fBB} , and t_{fCC} as the parameters. In the L-W mechanism, each regional government selects $0 \leq t_{fAA} \leq 1$, $0 \leq t_{fBB} \leq 1$, and $0 \leq t_{fCC} \leq 1$, which guarantee

$$d_A = d_{fAA}^D = d_{hAA}^D, d_B = d_{fBB}^D = d_{hBB}^D, d_C = d_{fCC}^D = d_{hCC}^D. \quad (17)$$

The selection of t_{fAA} , t_{fBB} , and t_{fCC} , however, must be done jointly with p_c and w , since in order to compute the minimum cost for providing d_A , d_B , and d_C , prices, p_c and w , must be known beforehand. Thus, p_c , w , t_{fAA} , t_{fBB} , and t_{fCC} as well as d_A , d_B , and d_C must be determined in the context of general equilibrium. The consumption good market is in equilibrium if the following equation holds.

$$x_{cAA}^S + x_{cBB}^S + x_{cCC}^S = x_{mAA}^D + x_{mBB}^D + x_{mCC}^D + x_{chAA}^D + x_{chBB}^D + x_{chCC}^D \quad (\text{National Market})$$

Labor market is in equilibrium if the following equation holds.

$$L_{0A} + L_{0B} + L_{0C} = l_{mAA}^D + v_{AA}^D + l_{mBB}^D + v_{BB}^D + l_{mCC}^D + v_{CC}^D + l_{cAA}^D + l_{cBB}^D + l_{cCC}^D$$

In the computation of general equilibrium with L-W mechanism, the Newton method is utilized with the normalization of $w=1$. This solution; $\{p_c^{*E}, t_{fAA}^{*E}, t_{fBB}^{*E}, t_{fCC}^{*E}, d_{AA}^{*E}, d_{BB}^{*E}, d_{CC}^{*E}\}$ is computed as follows.

$$\begin{aligned} p_c^{*E} &= 6.58045, t_{fAA}^{*E} = 0.347492, t_{fBB}^{*E} = 0.272914, t_{fCC}^{*E} = 0.233674, \\ d_{AA}^{*E} &= 124.792, d_{BB}^{*E} = 202.312, d_{CC}^{*E} = 275.966 \end{aligned} \quad (18)$$

Alternatively, $p_c^{*E}, t_{fAA}^{*E}, t_{fBB}^{*E}, t_{fCC}^{*E}, d_{AA}^{*E}, d_{BB}^{*E}$, and d_{CC}^{*E} can be computed by the following differential equations, where s is time.

$$\begin{aligned} dp_c[s]/ds &= x_{mAA}^D + x_{mBB}^D + x_{mCC}^D + x_{chAA}^D + x_{chBB}^D + x_{chCC}^D - (x_{cAA}^S + x_{cBB}^S + x_{cCC}^S) \\ dt_{fAA}[s]/ds &= d_{fAA}^D - d_{hAA}^D \\ dt_{fBB}[s]/ds &= d_{fBB}^D - d_{hBB}^D \\ dt_{fCC}[s]/ds &= d_{fCC}^D - d_{hCC}^D \\ dd_{AA}[s]/ds &= d_{hAA}^D - d_{AA} \\ dd_{BB}[s]/ds &= d_{hBB}^D - d_{BB} \\ dd_{CC}[s]/ds &= L_{0A} + L_{0B} + L_{0C} - (l_{mAA}^D + v_{AA}^D + l_{cAA}^D + l_{mBB}^D + v_{BB}^D + l_{cBB}^D + l_{mCC}^D + v_{CC}^D + l_{cCC}^D) \end{aligned} \quad (19)$$

Trajectory of and d_{AA} [s] on (19), are depicted as in what follows, which shows stability.

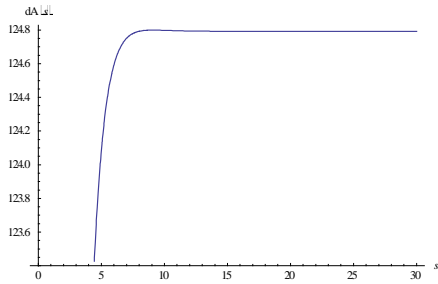


Fig. 3: Trajectory of d_{AA} [s]

We have local stability since all the real parts of eigenvalues for the Jacobian matrix at the equilibrium shows the negativity.

$$\begin{aligned} &-2545.65, -1624.64, -815.874, -14.7748, -1., -0.886252 + 0.126213 i, \\ &-0.886252 - 0.126213 i \end{aligned}$$

For the comparison with separate defense case, the regions' utility levels in the unified commodity

market, u_A^{*E} , u_B^{*E} , and u_C^{*E} , are computed.

$$u_A^{*E} = 35.9487, u_B^{*E} = 61.5204, u_C^{*E} = 86.1522 \quad (20)$$

The comparison between (8), (10), (12), and (20) shows that each of them, u_A^{*E} , u_B^{*E} and u_C^{*E} , is slightly greater than that for isolated case. However, the comparison between (15) and (20) shows

$$u_A^{*E} < u_A^{*C}, u_B^{*E} < u_B^{*C}, \text{ and } u_C^{*E} < u_C^{*C}$$

Thus, first, it is clear that general equilibrium for the market integration without military (political) integration is not Pareto optimal. In the present situation, the market integration without political integration is worse than the political integration without market integration. Note, however, that the latter is *not* Pareto optimal, either. There is a possibility to achieve the Pareto optimality by forming a nation: i.e. political integration and market integration.

Remark 1

In Fukiharu [2011], the author defined one of equilibrium conditions for two-region model, corresponding to (17), as in what follows without allowing labor migration.

$$d_{fAA}^D = d_{hAA}^D, d_{fBB}^D = d_{hBB}^D$$

By the simulation, he obtained $d_{fAA}^D = d_{hAA}^D = 114.376$, and $d_{fBB}^D = d_{hBB}^D = 190.31$. As is easily found, we have $d_A = 92.2261$ and $d_B = 212.46$. This discrepancy stems from the assumption of wage rate equality between the two regions in spite of the prohibition of labor migration. If we desire (17) as well as the prohibition of labor migration, the difference of the wage rates between the two regions must be introduced. Meanwhile, if the labor migration is allowed, (17) is guaranteed. In Fukiharu [2014a], this amendment was attempted. Allowing the labor migration, he obtained the following result.

$$d_{AA}^* = d_{fAA}^{D*} = d_{hAA}^{D*} = 114.376, d_{BB}^* = d_{fBB}^{D*} = d_{hBB}^{D*} = 190.31$$

Thus, he showed that if the labor migration is allowed, then, the desired defense levels in the previous simulation are guaranteed by the modification of equilibrium commodity price.

4. The Formation of A Nation with Defense Integration and Market Integration

We examine the formation of a nation, or the defense integration with national market for consumption good. The same price prevails in Regions A, B, and C for civilian goods. There is population of $L_{0A}+L_{0B}+L_{0C}$ in this nation. The production function, f_{1A} , of the civilian industries is stipulated by (1). Production function of the military industry, f_{2A} , is stipulated by (2). It is assumed that f_{2A} does not depend on d as before. Nation's definition of armed force, d , is stipulated by (3). The integrated government provides the level of the armed force by the minimum cost principle where the price of civilian good is p_c and the wage rate is w , on the assumption that the military personnel can be freely employed from Region A through C with the civilian wage rate. Furthermore, parameters on functions are stipulated by (4) and (5). Thus, the government computes the demand for civilian good, x_m^D , the demand for labor, l_m^D , and the demand for military personnel, v^D , with p_c , w , and d^0 parameters, solving the cost minimizing problem.

The consumption good industry in Region A also shares the burden of keeping the armed force. Suppose that t_{fA} is the share of burden for the consumption good industry in Region A. The behavior of the industry is stipulated by the profit maximization. From this maximization, demand function for labor, l_{cA}^D , the demand function for the armed force, d_{fA}^D , and supply function, x_{cA}^S , are derived with p_c , w , and t_{fA} parameters. The resulting (expected) maximum profit, π_A , is computed, which is distributed to the household in Region A.

By the same argument, demand function for labor, l_{cB}^D , the demand function for the armed force, d_{fB}^D , and supply function, x_{cB}^S , are derived with p_c , w , and t_{fB} parameters. The resulting (expected) maximum profit, π_B , is computed, which is distributed to the household in Region B. Following suit, in Region C, demand function for labor, l_{cC}^D , the demand function for the armed force, d_{fC}^D , and supply function, x_{cC}^S , are derived with p_c , w , and t_{fC} parameters. The resulting (expected) maximum profit, π_C , is computed, which is distributed to the household in Region C.

The (aggregate) household in Region A maximizes utility subject to income constraint. Thus, given p_c , w and t_{hA} , the household expresses demand function for consumption good, x_{chA}^D , and the demand function for the armed force, d_{hA}^D .

In the same way, given p_c , w and t_{hB} , the household in Region B expresses demand function for consumption good, x_{chB}^D , and the demand function for the armed force, d_{hB}^D . Following suit, given p_c , w and t_{hC} , the household in Region C expresses demand function for consumption good, x_{chC}^D , and the demand function for the armed force, d_{hC}^D .

So far, we have derived demand and supply functions with p_c , w , t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , as the parameters. In Lindahl-Walras mechanism, the integrated government selects $0 \leq t_{fA} \leq 1$, $0 \leq t_{fB} \leq 1$, $0 \leq t_{fC} \leq 1$, $0 \leq t_{hA} \leq 1$, $0 \leq t_{hB} \leq 1$, $0 \leq t_{hC} \leq 1$, $t_{fA} + t_{fB} + t_{fC} + t_{hA} + t_{hB} + t_{hC} = 1$, which guarantees

$$d^0 = d_{fA}^D = d_{fB}^D = d_{fC}^D = d_{hA}^D = d_{hB}^D = d_{hC}^D.$$

The selection of t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , however, must be done jointly with p_c , and w , since in order to compute the minimum cost for providing d^0 , prices; p_c and w , must be known beforehand. Thus, p_c , w , t_{fA} , t_{fB} , t_{fC} , t_{hA} , t_{hB} , and t_{hC} , as well as d^0 must be determined in the context of general equilibrium. The consumption good market is in equilibrium if the following equation holds.

$$x_{cA}^S + x_{cB}^S + x_{cC}^S = x_{mA}^D + x_{chA}^D + x_{chB}^D + x_{chC}^D.$$

Labor market is in equilibrium if the following equation holds.

$$L_{0A} + L_{0B} + L_{0C} = l_m^D + v^D + l_{cA}^D + l_{cB}^D + l_{cC}^D.$$

In the computation of general equilibrium with L-W mechanism, the Newton method is utilized with the normalization of $w=1$. This GE solution; $\{p_c^{*N}, t_{fA}^{*N}, t_{fB}^{*N}, t_{fC}^{*N}, t_{hA}^{*N}, t_{hB}^{*N}, t_{hC}^{*N}, d^{0*N}\}$ is computed as follows.

$$\begin{aligned} p_c^{*N} &= 4.52565, t_{fA}^{*N} = 0.143851, t_{fB}^{*N} = 0.0912627, t_{fC}^{*N} = 0.0912627, \\ t_{hA}^{*N} &= 0.143851, t_{hB}^{*N} = 0.242071, t_{hC}^{*N} = 0.34029, d^{0*N} = 628.71. \end{aligned} \quad (21)$$

Alternatively, the L-W solution, (21), can be derived by the following differential equations.

$$\begin{aligned} dp_c[s]/ds &= x_{mA}^D + x_{chA}^D + x_{chB}^D + x_{chC}^D - x_{cA}^S - x_{cB}^S - x_{cC}^S \\ dt_{fA}[s]/ds &= d_{fA}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dt_{fB}[s]/ds &= d_{fB}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dt_{fC}[s]/ds &= d_{fC}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dt_{hA}[s]/ds &= d_{hA}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dt_{hB}[s]/ds &= d_{hB}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dt_{hC}[s]/ds &= d_{hC}^D - (d_{fA}^D + d_{fB}^D + d_{fC}^D + d_{hA}^D + d_{hB}^D + d_{hC}^D)/6 \\ dd^0[s]/ds &= L_{0A} + L_{0B} + L_{0C} - (l_m^D + v^D + l_{cA}^D + l_{cB}^D + l_{cC}^D) \end{aligned} \quad (22)$$

The trajectory of $d^0[s]$ on (22) converge to (21) as shown by some of them in Fig. 4.

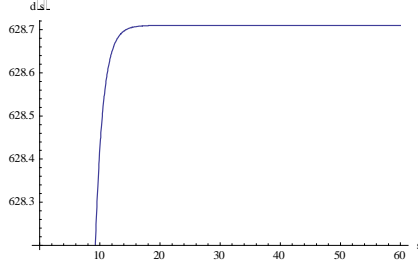


Fig. 4: Trajectory of $d^0[s]$

The eigenvalues of the Jacobian matrix of (22) at (21) are

$$-13778, -13778, -7706.85, -3630.94, -2181.14, -31.7025, -0.771191, 0$$

For the comparison with isolated defense case and the partial integration cases, the regions' utility levels in this nation, u_A^{*N} , u_B^{*N} , and u_C^{*N} , are computed as in what follows.

$$u_A^{*N} = 100.862, u_B^{*N} = 130.84, u_C^{*N} = 155.13 \quad (23)$$

The comparison between (20) and (23) shows that by the formation of a nation each region achieves higher utility level than that for isolated defense case. Thus, the formation of a nation is Pareto-improving compared with isolated defense case, which may well be one of feasibility conditions of coalition. It is examined next if the resource allocation in this nation is Pareto-optimal. If it is Pareto optimal, $u_A^{*N} = 100.862$ must be the maximal utility level for Region A, given the utility level for Region B at $u_B^{*N} = 130.84$, and the utility level for Region C at $u_C^{*N} = 155.13$. Thus, $u_A^{*N} = 100.862$ must be the solution to the following maximization.

$$\begin{aligned} & \text{Max } U_A[x_{cA}, d] \\ & \text{s.t. } U_A[x_{cB}, d] = u_B^{*N}, U_C[x_{cC}, d] = u_C^{*N}, d = f_{3A}[m, v], m = f_{2A}[x_m, l_m], x_m + x_{cA} + x_{cB} + x_{cC} = \\ & f_{1A}[l_{cA}, d] + f_{1B}[l_{cB}, d] + f_{1C}[l_{cC}, d], l_{cA} + l_{cB} + l_{cC} + l_m + v = L_{0A} + L_{0B} + L_{0C}. \end{aligned} \quad (24)$$

By the classical Lagrangian method, the maximal solution for (24) can be computed as in what follows.

$$\begin{aligned} x_{cA}^{NO} &= 16.1809, d^{NO} = 628.71, x_{cB}^{NO} = 27.2291, x_{cC}^{NO} = 38.2772, v^{NO} = 412.186, x_m^{NO} = 10.7031, \\ l_m^{NO} &= 48.4387, l_{cA}^{NO} = 46.4585, l_{cB}^{NO} = 46.4585, l_{cC}^{NO} = 46.4585. \end{aligned} \quad (25)$$

Remark 2

In the two-region case (Fukiharu [2012]), note that only $x_{cA}^{NO} \neq x_{cA}^{CO}$ and $x_{cB}^{NO} \neq x_{cB}^{CO}$ hold, while all the other optimizing values are the same.

The utility level under (25), u_A^{NO} , is computed as in what follows.

$$u_A^{NO} = 100.862 = u_A^{*N}. \quad (26)$$

Thus, the allocation in this nation is Pareto-optimal.

The comparison between (15) and (23), however, shows that the transition from the defense integration without market integration to the formation of a nation is *not* Pareto-improving. In other words, the transition from the isolated defense with isolated markets to the defense integration without market integration, then finally to the formation of a nation may not be a smooth process, since the first transition is Pareto-improving but the second transition is *not* Pareto-improving. Meanwhile, the comparison between (20) and (23) shows that the transition from the market integration without defense integration to the formation of a nation is Pareto-improving. In other words, the transition from the isolated defense with isolated markets to the market integration without defense integration, then finally to the formation of a nation may be a smooth process, since the first transition is Pareto-improving and the second transition is also Pareto-improving.

In the present-day world, Europe has followed the rational strategy in this paper, “first market integration, then, the defense integration”. The transition from the market integration to the defense integration is Pareto-improving as shown in this paper. Note, however, that this transition is not smooth in the real world. First of the reasons is that in this paper military industries are owned by the regional governments and their behavior is stipulated by the cost minimization for the provision of armed force. In the real world, the military industries may well be owned privately and behave so as to maximize profit. The second reason is that required reorganization of military industries in the defense integration is not easy task. This aspect is similar to the argument in the textbook argument for the free trade. In the elementary textbook, starting from an isolated (no trade) equilibrium to the trade equilibrium, “gains from trade” emerges. In this “gains from trade” argument, when the international price is lower than the isolated equilibrium price, producers’ surplus declines, even though consumers’ surplus increases. Producers may well oppose the introduction of trade, or demand protection by tariff. Even if the military industries are owned privately, the transition from market integration to the final defense integration may well be Pareto improving. However, one of the three military industries may well disappear in the reorganization. While theoretically this reorganization is easily done by definition, this reorganization may be a difficult task in the real world.

Remark 3

A suspicion that the conclusion in this paper might depend on the assumptions on production and utility functions as well as populations is cleared by another simulation, so long as the identical production and utility functions are assumed for all the regions. As an example, suppose that

$$\alpha_{1A}=\alpha_{1B}=\alpha_{1C}=1/6, \alpha_{2A}=\alpha_{2B}=\alpha_{2C}=2/3, \gamma_{1A}=\gamma_{1B}=\gamma_{1C}=3/4, \gamma_{2A}=\gamma_{2B}=\gamma_{2C}=1/4, n=1, \tau=-2/3, \beta_1=1/5, \beta_2=4/5, L_{0A}=100, L_{0B}=1000, \text{ and } L_{0C}=1100$$

We have the following result.

$u_A^*=40.3002, u_B^*=298.697, u_C^*=324.563$	Isolated (in Defense and Market) Case
$u_A^{*C}=230.458, u_B^{*C}=640.915, u_C^{*C}=648.597$	Defense Integration Case
$u_A^{*E}=47.8001, u_B^{*E}=299.021, u_C^{*E}=325.297$	Market Integration Case
$u_A^{*N}=149.477, u_B^{*N}=658.996, u_C^{*N}=702.77$	Formation of a Nation Case

For another example, suppose that

$$\alpha_{1A}=\alpha_{1B}=\alpha_{1C}=2/7, \alpha_{2A}=\alpha_{2B}=\alpha_{2C}=1/7, \gamma_{1A}=\gamma_{1B}=\gamma_{1C}=3/16, \gamma_{2A}=\gamma_{2B}=\gamma_{2C}=13/16, n=1, \tau=-1/4, \beta_1=1/9, \beta_2=8/9, L_{0A}=5000, L_{0B}=8000, \text{ and } L_{0C}=10000$$

We have the following result.

$u_A^*=2878.13, u_B^*=4215.93, u_C^*=5057.74$	Isolated (in Defense and Market) Case
$u_A^{*C}=8939.74, u_B^{*C}=10623.9, u_C^{*C}=10751.7$	Defense Integration Case
$u_A^{*E}=2912.4, u_B^{*E}=4216.28, u_C^{*E}=5076.74$	Market Integration Case
$u_A^{*N}=9836.33, u_B^{*N}=10535.5, u_C^{*N}=10909.4$	Formation of a Nation Case

These results guarantee the conclusion in this paper. When the identity assumption is dropped, however, the conclusion in this paper is not guaranteed. As an example, suppose that

$$\alpha_{1A}=1/6, \alpha_{1B}=3/5, \alpha_{1C}=1/2, \alpha_{2A}=2/3, \alpha_{2B}=1/4, \alpha_{2C}=1/3, \gamma_{1A}=2/5, \gamma_{1B}=2/3, \gamma_{1C}=5/6, \gamma_{2A}=3/5, \gamma_{2B}=1/3, \gamma_{2C}=1/6, n=1, \tau=-2/3, \beta_1=1/6, \beta_2=5/6, L_{0A}=100, L_{0B}=200, \text{ and } L_{0C}=300$$

We have the following result.

$u_A^*=62.5223, u_B^*=78.3935, u_C^*=86.2945$	Isolated (in Defense and Market) Case
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$u_A^{*C}=175.635, u_B^{*C}=131.718, u_C^{*C}=123.588$	Defense Integration Case
$u_A^{*E}=83.0748, u_B^{*E}=64.4472, u_C^{*E}=84.0171$	Market Integration Case
$u_A^{*N}=192.778, u_B^{*N}=132.449, u_C^{*N}=158.851$	Formation of a Nation Case

This result does not guarantee the conclusion in this paper, since the transition from the isolation to the market integration is *not* Pareto-improving, although the transition from the market integration to the formation of a nation is Pareto-improving. Meanwhile, the transition from the isolation to the defense integration is Pareto-improving, *and*, the transition from the defense integration to the formation of a nation is also Pareto-improving. Thus our conclusion depends on the *identity* assumption of production and utility functions. (The computation in Remark 3 was conducted in Fukiharu [2014c].) This analysis is similar to Fukiharu [2004], who asserted that the Heckscher Ohlin Theorem depends on the *identity* assumption of production and utility functions of the two trading countries.

Conclusions

The aim of this paper is to examine European countries' strategy of becoming one nation politically as well as economically. After the two great wars, devastating their own territories, they have attempted to become one society, by removing the walls of present countries. For this sincere effort, in 2012, the European Union, EU, was awarded the Nobel Prize for Peace. In this paper, defining their effort as the strategy of "first the integration of (isolated) markets, then the integration of (isolated) defense: the formation of one nation politically and economically", we compare it with another strategy of "first the integration of (isolated) defenses, then the integration of (isolated) markets: the formation of one nation politically and economically", by constructing a primitive general equilibrium model. It is possible to examine regional and national defense from the viewpoint of public good in economics. The optimal defense level can be computed through Lindahl method, and the approach in this paper is defined the Lindahl-Walras general equilibrium.

We start with the examination of three regions, A, B, and C, facing a common intruder. It is assumed that their production and utility functions are identical among the three regions, with different populations. These regions suffer from the destruction of production facilities for consumption good. By introducing regional defense they can raise their production of consumption good, as well as household's utility through the reduced threat. In the provision of this defense, military good and personnel are required. In this paper, the provision of defense is made by (regional or national) government through the minimum cost principle. The firm and household bear the cost for this provision through Lindahl method, since defense is a public good. When three regions are isolated from each other, they have their own regional markets for consumption good and different

defense level. In this paper, a simulation approach is adopted, with parameters on the production and utility functions etc. as well as the populations specified numerically. It is possible to compute general equilibrium and regions' utility levels.

In the second, we examine the case of defense integration (without market integration) of Regions A, B, and C. In this integration, it is assumed that the allied government provide the defense for all the regions, by the Lindahl method. Different commodity prices emerge, since markets are not integrated. Note, however, that labor migration among three regions is allowed in this paper. In the general equilibrium, each region's utility level is higher than the corresponding one in the completely isolated case. Thus, Pareto improvement is realized. It is shown that this case is not Pareto optimum.

In the third, we examine the case of market integration (without defense integration) of Regions A, B, and C. In this integration, each regional government provide the defense for its isolated region, by the Lindahl method. The identical commodity price emerges, since markets are integrated. Labor migration among three regions is also allowed. In the general equilibrium, each region's utility level is higher than the corresponding one in the completely isolated case. Thus, Pareto improvement is also realized in this case. It is shown also that this case is not Pareto optimum.

In the fourth, we examine the case of defense integration and market integration of Regions A, B, and C: formation of a nation. In this integration, it is assumed that the allied government provide the defense for all the regions, by the Lindahl method. The identical commodity price emerges, since markets are integrated. In the general equilibrium, each region's utility level is higher than the corresponding one in the completely isolated case. Thus, Pareto improvement is realized. Furthermore, it is shown that Pareto-optimum is realized in this case. In order to examine the transition from the second case to the fourth, we compare the utility variation in this transition. It is shown that some of the utilities in the second case are lower than those corresponding ones in the fourth case. Thus, this transition is not Pareto-improving. When we examine the transition from the third case to the fourth case, it is shown that all of the utilities in the third case are higher than those corresponding ones in the fourth case. Thus, this transition is Pareto-improving.

The conclusion, asserted so far, was shown to depend on the assumption on the identity of production and utility functions of three regions. If the different functions are selected, it was shown that the assertion does not hold. Thus, from the purely theoretical examination, the present policy adopted by the European countries: "first the integration of markets, next the integration of defense", is feasible and rational, so long as the difference of production and utility functions among European countries is sufficiently small. .

Acknowledgements

The present author appreciates financial support from the JSPS KAKEN Grant Number 25380235.

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