Repeated Burden Sharing and Costly Negotiations
Preliminary Version

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Abstract

In a setting of two parties repeatedly facing a public good problem with risky benefits, I study the option of a non-binding burden sharing rule as a way to avoid frequent negotiations that are costly. Under risk neutrality, I find such a rule to be attractive if parties are patient enough, and its design to match the expected negotiation outcome independent of individual benefit variance. Under risk aversion, however, the contract is less attractive as it reduces the scope for risk sharing. For asymmetric variance, the rule affects risk allocation and may deviate from the expected outcome.

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1 Introduction

By nature, a public good that requires financial investment poses the question of who contributes how much. Negotiations between the parties that stand to benefit from the public good may be necessary to determine the answer to this question. One can view the outcome to consist of two elements, or dimensions: (i) total investment, or the sum of all contributions, and (ii) the share of this total that is contributed by each individual party.

The size of the burden and the way it is shared can be negotiated either jointly or separately. Whichever way applies, successful negotiations should result in specified contributions for each party and provision of the public good for everyone's benefit.

Many public goods cannot be consumed for ever after a one-time investment, but repeatedly require investments for their provision to be continued. Accordingly, parties might negotiate contributions whenever new investment is needed. As time goes by, changing circumstances can affect the outcome these negotiations. Examples of where change

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might occur are one's financial capacity, the available alternatives, how costly the public good is or how much one expects to benefit from its provision.

Despite continuous developments that can shift the relative bargaining positions, we generally do not renegotiate public good contributions at every single opportunity. This seems to be the case especially for the second dimension mentioned above, that of how to divide the burden. Even burden sharing rules that are not legally binding seem to be quite stable, such as those that are internationally agreed between national governments.

The funding structures of most international development banks are examples of this. These banks are public institutions that are funded by member countries providing capital. From time to time the members decide these banks need a capital injection. The associated negotiations then usually only concern the size of the overall increase in capital, and to what aim it should be put to use, while the contribution per member state is proportional to the existing division of shares.

Another example is the EU budget, which is determined annually in terms of expenditures, constrained by ceilings that are determined in the Multi-annual Financial Framework. The lion's share of the required revenue comes from direct member state transfers that are set as a percentage of each member's GNI, thereby ignoring change in all other relevant economic, financial and political variables. Only very occasionally are individual exceptions negotiated. [1]

One possible explanation for burden division not being renegotiated at every opportunity is that these negotiations are costly, and the costs are not outweighed by the benefits. Negotiation costs can be interpreted as the time and effort agents have to spend, reputation costs or reduced benefits resulting from the provision uncertainty that is associated with renegotiations.

When negotiations are costly, a non-binding agreement that specifies a rule for burden sharing can be an attractive option. Even when not legally binding the agents, such an agreement specifying default contributions can prevent them from having to negotiate every single period.

This paper offers an initial analysis of such burden sharing rules, by examining a setting in which two parties repeatedly face a burden sharing problem, while negotiations are costly. At the start, the parties have the option to negotiate a rule, without perfectly foreseeing their bargaining positions in future periods. The framework is used to investigate the effect of several factors, both on the value of the burden sharing rule and on the contributions it specifies.

The analysis shows that the value of having a burden sharing rule is positively related to the parties' discount factor and to the cost of negotiations. Whenever surplus is large

[1] This may happen more extremely and with higher frequency in coming years, given the Brexit-related instability.

relative to negotiation cost, the rule produces fixed contribution levels whereas per-period negotiating means they adjust to bargaining positions. The value of the rule then lies in the negotiation costs that are avoided whenever the rule is honored. When surplus is relatively small, however, the public good provision itself depends on the rule. In that case, the rule's value comes from the realized public good benefits.

While a non-binding rule can enhance welfare by offering a way around costly negotiation, the analysis shows it also increases individual payoff variation and can affect the allocation of risk between two parties. The underlying reason for this is that whenever the contract is honored, shocks at the individual level are fully absorbed by one party, while under per-period negotiations individual shocks feed into both parties' payoffs. For risk-averse parties, this reduces the value of the burden sharing rule. When the risk in individual public good benefit is asymmetric, risk aversion can push the default contributions away from the expected (re)negotiation outcome.

There is a small literature on costly negotiation that relates to the analysis presented here, although not in a setting involving equivalent parties and a public good problem. One example is the paper by McCutcheon (1997), which shows how the prohibition of price agreements and the negotiation thereof may facilitate rather than prevent price collusion, when firms are patient enough and the effective costs of negotiations are within a certain range.

Other authors have examined buyer-seller settings where parties face uncertainty about value and cost. Although a different setting, this similarly comes down to parties generating and dividing surplus. Hart (2009)[2] looks at contracts as reference points, on which parties can force renegotiations. If that occurs, welfare loss is created as one or both parties feel they do not get what they are entitled to, resulting in a hostile relation. These contracts are similar to what will be analyzed here, in that they define a range of realized states for which they will not be renegotiated. The main difference is that while both contracts, or agreements, are to an extent non-binding, the type considered here is an option that offers a way to avoid costly negotiations, while that considered by Hart (2009) generates a cost when deviated from.

Perhaps most similar is the work by Masten (1988), who studies the impact of transaction costs on contract design. Masten shows how the cost of renegotiations determine whether or not buyer or seller initiate them. The work shows that one objective in structuring a contract is equating, on the margin, the expected costs of opportunistic behavior on both sides of the relation. This corresponds to our findings for similar scenarios.

While Hart (2009) and Masten (1988) focus on the welfare-maximizing contract, they do not consider the contract itself as a bargaining outcome, which is a feature in the model presented here. Also, the mentioned papers do not analyze the effect a contract has on the allocation of risk. The findings on that subject relate to the work of Perloff

[2]Building on Hart and Moore (2008)

(1981), which considers under what circumstances breaches in forward contracts should be excused by the court. Perloff analyzes transactions between risk-averse farmers and risk-neutral buyers and shows how reducing the range in which the contract is effectuated impacts the income variation of the farmers. Allowing contract breach under extreme circumstances is shown to have an ambiguous effect on income variance and thereby on welfare, depending on the correlation between price and harvest. The case analyzed here involves both sides of the relation having similar risk preferences, finding that creating some contract range unambiguously increases payoff variation, counteracting the positive effect of reducing negotiation frequency.

The paper is organized as follows: Section 2 lays out the model setup, Section 3 studies how the outcomes depend on several parameters under varying assumptions, Section 4 concludes.

2 The Model

Consider a setting with two parties $i \in \{A, B\}$, who have a joint interest in the provision of a public good. Time is modeled in discrete periods $t \in \{0, 1, 2, ..., T\}$. In each period, starting at $t = 1$, the parties can contribute to the public good by investing $q_{i,t} \geq 0$. The contribution effects are assumed to be symmetric, additively separable and linear. Their payoffs in each period depend on the sum of their contributions and on their individual benefit factors h_t^i:

$$\pi_t^i = h_t^i Q_t - q_t^i \quad \text{with} \quad Q_t = \sum_i q_t^i. \tag{1}$$

Capacity is assumed to be constrained at a maximum of one, and attention is restricted to situations where the individual benefits from the public good do not outweigh its cost:

$$Q_t \leq 1 \tag{2}$$

$$h_t^i < 1 \quad \forall \ i. \tag{3}$$

For each party, h_t^i is independently drawn every period from a commonly known distribution funtion $F_i(\cdot)$ with density function $f_i(\cdot)$, mean μ_i and standard deviation σ_i. These benefit factors become common knowledge shortly before period t, say at date t^-.

In each period $t \geq 1$, if both parties are willing, they can enter negotiations to determine cooperative contribution levels $\boldsymbol{q_t} = \{q_t^A, q_t^B\}$ for that period. I assume that once agreement is reached on cooperative contribution levels, the parties do not cheat each other by deviating from $\boldsymbol{q_t}$.

Negotiating is costly, with a symmetric cost α incurred on both parties, that is unrelated to the outcome. After negotiations in period t, the payoff of party i in that period is therefore given by

$$\pi_t^i = h_t^i Q_t - q_t^i - \alpha. \tag{4}$$

In period $t = 0$, the parties can negotiate a burden sharing rule (BSR), specifying standard contribution levels $\bar{\boldsymbol{q}} = \{\bar{q}^A, \bar{q}^B\}$. In all subsequent periods, they can contribute according to this BSR, without having to engage in costly negotiations. The BSR is, however, not legally binding: In case either of them wishes to opt out of the rule for a particular period, the parties will revert to non-cooperative contribution levels, unless they are both willing to renegotiate (at cost α) cooperative contributions for that period.

The rule will thus only be effectuated if both parties choose to honor it. Agreeing on a BSR at $t = 0$ therefore does not take away any of the options the parties have in the subsequent periods, it simply adds a possible outcome that does not require costly negotiations. For all periods $t \geq 1$, the BSR will be in place regardless of the actions and outcomes of previous periods.[3]. In the following, contribution levels and the BSR are determined by backwards induction.

2.1 Contributions in period $t \geq 1$

The parties are assumed to employ Markovian strategies, in the sense that they only base their decisions on payoff relevant information, i.e. on $\bar{\boldsymbol{q}}$ and on the realized benefit factors $\boldsymbol{h}$.[4] Under this assumption, all periods $t \geq 1$ can be treated as ex ante identical.

2.1.1 Non-cooperative

First consider the non-cooperative case in which there is no agreement on contribution levels (by one-time negotiations nor by BSR). The parties will then determine their contributions simply by maximizing (1). Denote non-cooperative contribution in period t as q_t^N. As long as (3) holds, they will both contribute nothing:

$$\max_{q_t^i} (q_t^i + q_t^j) h_t^i - q_t^i \iff q_t^N = 0.$$

This implies non-cooperative payoff is zero for both parties: $\pi_t^N = 0$.

[3]Even if the parties have the option to renegotiate not just period t-contributions but also the BSR itself, they would not do so. In the setting examined here, the payoff functions are history-independent, so at any date $t > 0$ there is no new information about future payoffs, compared to $t = 0$, that might motivate either party to renegotiate $(\bar{q}^A, \bar{q}^B)$.

[4]If the parties did not agree on a BSR in $t = 0$ (i.e. did not enter BSR negotiations or failed to reach agreement), one can think of the BSR as $\bar{\boldsymbol{q}} = \{0, 0\}$.

2.1.2 Cooperative

These zero contribution levels serve as the threat point for negotiations on $\boldsymbol{q_t}$. Without a BSR or in a period where either party opts out of the BSR, positive contributions will require negotiations. I assume that when they negotiate, the parties maximize the (symmetrically weighted[5]) Nash product of their respective payoff gains relative to the threat point:

$$\max_{q_t}(h_t^A - q_t^A - \pi_t^N) \times (h_t^B - q_t^B - \pi_t^N)$$

Note that negotiation cost α does not enter here because it is incurred regardless of whether or not the negotiations are successful. The contributions $\boldsymbol{q_t}$ they will agree on are described in the following Lemma.

Lemma 1. *If the sum of the realized benefit factors exceeds the marginal contribution effect (=1), Nash Bargaining results in the parties jointly investing up to the capacity constraint (and is zero otherwise). Individual contributions depend on the relative public good benefits, and the resulting payoffs are equal for both parties:*

$$Q_t = 1 \qquad \text{if} \qquad h_t^A + h_t^B > 1 \tag{5}$$

$$q_t^A = \frac{1}{2} + \frac{h_t^A - h_t^B}{2}, \tag{6}$$

$$q_t^B = \frac{1}{2} - \frac{h_t^A - h_t^B}{2}, \tag{7}$$

$$\pi_t^A = \pi_t^B = \frac{h_t^A + h_t^B - 1}{2} - \alpha. \tag{8}$$

The proof is given in the Appendix. The cooperative payoffs show that the parties split the surplus evenly. Accordingly, the difference between individual contributions matches that in realized benefit factors. If the condition in (5) does not hold, contribution levels will be zero. It can therefore be interpreted as a requirement for successful negotiations.

Having established that negotiations will always result in $Q_t = 1$, we can treat the cooperative contributions as one-dimensional, denoting the contribution by A as q_t:
$\boldsymbol{q} = \{q_t^A, q_t^B\} = \{q_t, 1 - q_t\}$.

2.1.3 Condition for entering negotiations

Willingness to negotiate requires the benefits to outweigh the costs, where it is assumed the parties perfectly foresee the outcome. For symmetric negotiation costs α, A and B

[5]Subsection 3.2.4 examines the case of asymmetric bargaining strength.

will make the same decision on whether or not to negotiate.

Lemma 2. *For positive α, parties enter negotiations iff the following condition*[6] *is satisfied:*

$$h_t^A + h_t^B \geq 1 + 2\alpha. \tag{9}$$

This follows directly from (8), as cooperative payoffs turn negative when (9) is not satisfied. The condition for entering negotiations is thus stronger than the condition for them being successful. An obvious interpretation of (9) is that the probability of not cooperating, while mutually beneficial in itself, is increasing in α.

2.1.4 Non-binding Burden Sharing Rule

With a BSR in place, the parties individually decide each period whether or not to opt out, after observing the realizations of $\boldsymbol{h}$. If either decides to do so, what happens next is as described in the previous subsections. Note that as the BSR is not binding, it does not affect the threat point of any potential renegotiations, the outcome of which is therefore still as stated in Lemma 1.

Renegotiating is costly, but opting out and reverting to non-cooperative contributions is not. Any outcome where at least one party has negative payoff can therefore be ruled out, which means any BSR will only be effectuated if the condition on joint benefits given in (5) is satisfied. Accordingly, any pareto optimal BSR must specify contribution levels that sum up to $Q_t = 1$. As such, we can also treat the BSR contributions $\bar{\boldsymbol{q}}$ as one-dimensional, denoting the contribution by A as $\bar{q}$:
$\bar{\boldsymbol{q}} = \{\bar{q}^A, \bar{q}^B\} = \{\bar{q}, 1 - \bar{q}\}$.

Each party effectively faces a choice between two payoffs, that depend on the realizations of $\boldsymbol{h}$ and on $\bar{q}$. If both choose to honor the BSR, negotiation costs are avoided and payoffs are

$$\bar{\pi}_t^A = h_t^A - \bar{q}, \tag{10}$$

$$\bar{\pi}_t^B = h_t^B - (1 - \bar{q}). \tag{11}$$

The alternative payoff depends on $\boldsymbol{h}$. If condition (9) is not satisfied, renegotiation will not take place and it is straightforward to see that a party will not opt out as long as $\bar{\pi}_t^i \geq 0$. If (9) does hold, the alternative payoffs are given by (8), and the payoff-maximizing choices depend on the difference in benefit factors, as is described by Lemma 3.

[6] We assume that, when indifferent, the parties will opt to negotiate and provide the public good.

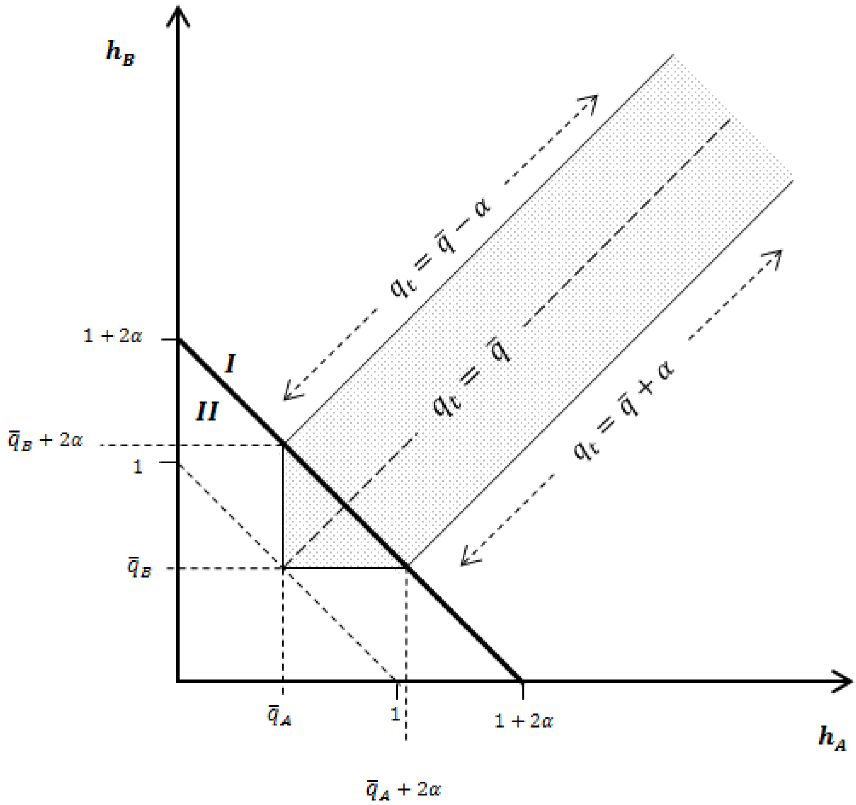

Figure 1: The Contract Range

Lemma 3. *In any period where the joint benefit from the public good is above the threshold of* $1 + 2\alpha$*, neither party will trigger renegotiations if the difference in benefit factors falls in the following range*[7]*, defined by* $\bar{\boldsymbol{q}}$*:*

$$2\bar{q} - 1 - 2\alpha \leq h_{A,t} - h_{B,t} \leq 2\bar{q} - 1 + 2\alpha. \tag{12}$$

This condition is equivalent to $\bar{\pi}_t^i \geq \pi_t^i$ for both i. It shows a range in relative benefit factors that determine the parties' respective bargaining positions. If the draws of benefit factors result in an unusually strong bargaining position for one of the parties, it will opt out of the BSR to exploit this position. This is more directly reflected when the range is rewritten in terms of the burden sharing q_t that would result from renegotiations:

$$q^L = \bar{q} - \alpha \leq q_t \leq \bar{q} + \alpha = q^H. \tag{13}$$

Here, q^L and q^H are the boundary levels of q_t for which the agreement is honored, and they are further apart the higher negotiation cost α.

[7] I assume that, when indifferent, the parties stick to the agreement.

Given the BSR, we now know whether or not the agreement will be honored for any combination of benefit factors $\{h_t^A, h_t^B\}$. Figure 3 graphically illustrates this. Any point in area *I* satisfies condition (9), while any point in area *II* does not. The shaded area represents the range in which the contract will be honored, and can be shifted in north-western or south-eastern direction by choice of $\bar{q}$. In area *II* it reflects that both parties need to have benefit factors above their BSR-contributions. In area *I*, it are the relative values that should not deviate too far from the line that represents $q_t = \bar{q}$.

2.2 Negotiating the BSR at $t = 0$

The analysis so far has shown how payoffs in $t \geq 1$ will depend on $\bar{q}$, which is negotiated at $t = 0$. The parties determine $\bar{q}$ by maximizing the Nash product of the gain in expected payoffs generated by the BSR. The threat point for these negotiations is therefor no BSR, or $\bar{\boldsymbol{q}} = \{0, 0\}$.

2.2.1 Expected BSR value

To determine the outcome of the BSR negotiations, it is helpful to define its expected value. Denote V_t^i as the difference in payoff, for party i in period t, between having and not having a BSR. As payoffs are not affected when contributions are renegotiated or canceled, the agreement will have zero value if not honored. When both parties decide to stick to the BSR, its value depends on the benefit factors. If (9) holds and renegotiations would be successful, we have

$$V_t^i = \frac{h_t^i - h_t^j + 1}{2} + \alpha - \bar{q}^i. \tag{14}$$

Note that this is never negative for both parties since (12) holds, and that the sum of the individual values is equal to the avoided negotiation costs, $V_t^{A+B} = 2\alpha$. If (9) does not hold, we have

$$V_t^i = h_t^i - \bar{q}_t^i, \tag{15}$$

which again is never negative when neither party opts out. The sum of individual values in this case is $V_t^{A+B} = h_t^A + h_t^B - 1$.

The parties know the probability of any realization of $\boldsymbol{h}$, and can thereby form expectations about V_t^i as a function of $\bar{q}$. As all periods $t > 0$ are ex ante the same, the expected value of the BSR at $t = 0$ for party i , as $T \to \infty$, is[8]

$$\frac{\delta}{1 - \delta} EV_i, \tag{16}$$

[8] δ is the discount factor and is assumed to be identical for both parties. As the BSR is determined in $t = 0$ and its value materializes only in later periods, any BSR that generates positive expected value will be more attractive the higher the discount factor.

where $EV^i = E_0[V_t^i(\bar{q})]$. These values are relevant, in relative terms, to determine the outcome of Nash Bargaining on the BSR. Furthermore, it shows how attractive such a rule is to both parties, which may determine whether or not it is established at all. This is the case if there is some threshold BSR-value below which the parties decide they do not negotiate one, for example because of costly negotiations at $t = 0$ [9].

2.2.2 BSR Bargaining Solution

The resulting BSR maximizes the Nash product[10] of its value to A and B:

$$\underset{\bar{q}}{\text{Max}} \quad (EV^A) \times (EV^B), \tag{17}$$

which gives the first-order condition

$$\frac{\partial EV^A}{\partial \bar{q}} EV^B = -\frac{\partial EV^B}{\partial \bar{q}} EV^A. \tag{18}$$

The following section will characterize this solution and examine how it is affected by various parameters and assumptions.

3 Results

The analysis here will focus on a scenario where the public good provision is always worth the required investment and negotiations ($h_{A,t} + h_{B,t} \geq 1 + 2\alpha$). This means the realizations of the random variables will always correspond to area I in figure 3. While a full analysis without this assumption is beyond this paper, a specific case with fully correlated benefit factors will be presented in subsection 3.6.

3.1 Baseline

The choice of $\bar{q}$ affects BSR values through two channels. First, it determines the individual payoffs whenever the rule honored. Second, it determines the probability of the BSR being honored. Both channels are relevant for the bargaining solution.Not surprisingly, it turns out the resulting BSR specifies contribution levels that match the ex ante expected outcome of negotiations in any single period.

[9] We could arbitrarily set such a cost, say α_0, that may or may not be equal to α. This does not, however, affect the outcome of the negotiations.

[10] Note that without the possibility for transfers at $t = 0$, the time preferences have no effect on the bargaining solution as they are factored out of the Nash product. (17) is therefore equivalent to $\underset{\bar{q}}{\text{Max}} \quad (\frac{\delta}{1-\delta} EV^A) \times (\frac{\delta}{1-\delta} EV^B)$

Proposition 1. *As long as distributions F_A and F_B are symmetric around their mean, the BSR reflects the expected negotiation outcome, depending only on the mean benefit factors. The design of the BSR does not depend on the variances in benefit factors:*

$$\bar{q} = \frac{\mu_A - \mu_B + 1}{2} = E[q_t]. \tag{19}$$

The attractiveness of the BSR to both parties depends only on $\sigma_A^2 + \sigma_B^2$, not on individual variance.

Appendix A2 contains the complete proof, some parts of which are described in the following.

We have two random variables h_t^A and h_t^B that for each $\bar{q}$ jointly determine whether or not renegotiations are triggered. This is captured by equation (6), which defines q_t as a function of the benefit factors, and the 'BSR-range' given by (13). Furthermore, we can rewrite (14) to obtain the per period BSR values to each party as a function of q_t and $\bar{q}$:

$$V_t^A = q_t - \bar{q} + \alpha \tag{20}$$

$$V_t^B = \bar{q} - q_t + \alpha$$

Using the cooperatively determined expression for q_t, we can define a distribution F_q over q_t that maps h_t^A and h_t^B into $F_q(q_t)$:

$$q_t = \frac{1}{2} + \frac{h_t^A - h_t^B}{2}, \tag{21}$$

The expected BSR values can then be expressed as

$$EV^A = \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} q_t - \bar{q} + \alpha \;\; dF_q(q_t), \tag{22}$$

$$EV^B = \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} \bar{q} - q_t + \alpha \;\; dF_q(q_t).$$

This allows us to determine the $\bar{q}$ that satisfied the first-order condition given by (18). For any distribution F_q that is symmetric around its mean, which is the case as long as F_A and F_B have that same property, we find:

$$\bar{q} = \mu_q = \frac{\mu_A - \mu_B + 1}{2} = E[q_t], \tag{23}$$

which implies

$$f_q(\bar{q} + \alpha) = f_q(\bar{q} - \alpha). \tag{24}$$

The BSR thus equates the probability density at the boundaries of the BSR-range, thereby minimizing the probability of renegotiations (for any single peaked distribution). At this

$\bar{q}$, the individual BSR values for both parties are equal to the expected value of the avoided negotiation costs:

$$EV^A = EV^B = \alpha \int_{\mu_q - \alpha}^{\mu_q + \alpha} dF_q(q_t).$$

This solution does not depend on the variance of the benefit factors. That is not to say the variance in one or both benefit factors is irrelevant, as it negatively affects the probability that renegotiation is avoided. It is this probability and the avoided negotiation costs α that determine the attractiveness of the BSR. For the off-contract outcome q_t, we know from its construction in (6) that its variance must be

$$\sigma_q^2 = \frac{\sigma_A^2 + \sigma_B^2}{4}. \tag{25}$$

An increase in variance on either party's benefit will therefore symmetrically reduce both parties' BSR value. Furthermore, (24) reveals that this value depends only on the sum of the variances, not on the variance in individual benefits.

3.1.1 Correlated Benefits

When h^A and h^B are not completely independent, their correlation affects the value of the BSR, as it changes the variance in q_t. When positively correlated, the benefit factors will 'move together'. As it is their realized difference relative to their expected difference that determines whether or not one of the parties will want to renegotiate contributions, positive correlation will increase the probability of the BSR being honored.

To see this, we can look at how correlation affects the variance of q_t. For

$$q_t = \frac{1}{2} + \frac{h_{A,t} - h_{B,t}}{2},$$

and correlation ρ, we have

$$\sigma_q^2 = \frac{\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B}{4}. \tag{26}$$

So a positive correlation between h_A and h_B ($\rho > 0$) reduces the variance of q_t and thereby increases the expected value of the contract to both parties. A negative correlation would, of course, have the opposite effect. This does not, however, affect the design of the BSR that results from bargaining.

3.2 Risk Aversion

For the baseline case the BSR simply reflects the expected future bargaining outcome, and its only economic effect is that it allows the parties to avoid costly negotiations each period. There are, however, effects on the total level and allocation of risk.

Without a BSR, parties negotiate contributions every period, and thereby fully share the variance. The surplus in every period is split 50:50, and payoffs are as given by (8). The variance of these individual payoffs is the same as the variance of the (re)negotiation outcome, $\frac{\sigma_A^2+\sigma_B^2}{4}$.

For any period that involves BSR contributions, individual payoffs only move with individual benefits. In the extreme case where parties always pay a fixed contribution and renegotiations are ruled out, the variance in the payoff π^i of party i would be equal to the variance in the individual benefit factor, σ_i^2.

The difference between these opposite cases is informative. The BSR examined in this paper shifts the situation from the prior, i.e. fully flexible contributions, in the direction of the latter, i.e. fixed contributions, affecting payoff risk in two ways. First, it increases total payoff variance, by preventing risk sharing within the BSR-range. Second, it affects the allocation of this total variance, as party i's own benefit factor variance will carry more weight in determining i's individual payoff variance. This means that differences in the σ_i^2's will be reflected in the variance of individual payoffs.

When parties care not only about expected payoff, but also dislike payoff variation, these effects have to be taken into account. The increase in total payoff variance is then an unattractive aspect of the BSR. At the same time, for asymmetric benefit variances, the party with the higher benefit variance will take on more payoff risk under the BSR. Risk aversion then reduces (relative to risk neutrality) the value of the BSR at $t = 0$ asymmetrically, which alters the bargaining positions, leading to the following conjecture.

Conjecture 1. *When $\sigma_A^2 < (>)\sigma_B^2$, and the parties are risk averse, the BSR, relative to the expected negotiations outcome $E[q_t]$, specifies lower contributions for A(B).*

In the following I will examine the special case where one of the parties has a fixed, certain benefit factor.

Assume that party B now has a fixed benefit factor $\hat{h}^B$, while h_t^A varies. To allow for risk aversion, let i's utility in t be a (common) function u of π_i, which we have defined as a linear combination of its contribution, benefit factor and potential negotiation cost.

$$u_t^i = \begin{cases} u(\pi_t^i) = u(h_t^i - q_t^i - \alpha) & \text{outside BSR-range} \\ u(\bar{\pi}_t^i) = u(h_t^i - \bar{q}^i) & \text{within BSR-range} \end{cases} \tag{27}$$

where $u' > 0$, $u'' < 0$ and $u''' \geq 0$.

The per-period negotiation outcome q_t is no different than under risk-neutrality, as all uncertainty about the benefits resolves before those negotiations commence:

$$q_t = \frac{1}{2} + \frac{h_t^A - \hat{h}^B}{2}, \tag{28}$$

The corresponding expected BSR values are based on the difference between the utilities in (27):

$$EV^A = \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u(\bar{\pi}_t^A) - u(\pi_t^A) \ dF_A(h_t^A), \tag{29}$$

$$EV^B = \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u(\bar{\pi}_t^B) - u(\pi_t^B) \ dF_A(h_t^A).$$

If we now set the BSR according to $\bar{q} = \frac{\mu_A - \hat{h}_B + 1}{2} = E[q_t]$, and differentiate with respect to $\bar{q}$, we obtain a negative value for the derivative (see Appendix 3). This indicates that the bargaining solution requires $\bar{q}$ to be lower than $E[q_t]$.

The BSR will therefore specify a higher contribution by party B than would be the case under risk-neutrality. In all $t \geq 1$ where the BSR is effectuated, B then has a certain payoff, while A's payoff absorbs all the risk.

3.3 Allowing for Smaller Surplus

So far the sum of the benefit factors has been assumed to not drop below $1+2\alpha$, guaranteeing public good provision, whether through BSR- or renegotiated contributions. For this range of benefit factors, the main value of the BSR lies in avoiding negotiation costs. For lower benefits, however, it could facilitate the public good provision itself, when the cost of negotiations would otherwise prevent the parties from starting them and cause them to default to zero investment.

To analyze this aspect, I consider variance in joint benefit rather than in relative bargaining positions. Assume the parties' benefit factors are subject only to a common shock, and are thereby fully correlated:

$$h_t^i = \hat{h}^i + \theta_t, \tag{30}$$

where $\hat{h}^i$ is a constant individual benefit and θ_t refers to the common shock in period t. If we assume the latter to have a distribution F_θ, with a zero mean, we can say that $\hat{h}^i$ is the mean individual benefit for i.

As deviations from their means are simultaneous, the difference between benefit factors is constant over time. The relative bargaining positions will therefore not change from period to period. Accordingly, the (re)negotiating contribution levels for one period will always have a certain outcome:

$$q_t = \frac{\hat{h}^A - \hat{h}^B + 1}{2}. \tag{31}$$

With q_t not varying, we know that for any $\bar{q}$ that satisfies

$$q_t - \alpha \leq \bar{q} \leq q_t + \alpha, \tag{32}$$

neither party will be able to improve their payoff by entering costly renegotiation in any period t. Any such BSR would only be canceled when either or both parties have a benefit lower than their respective contribution.

The value V_t^i of the BSR in period t depends on the size of the surplus, as described by equations (14) and (15). For larger surplus, the rule avoids costly negotiation. For a positive surplus smaller than 2α, the BSR is necessary for the public good to be provided at all.

The individual expected values again depend on the design of the BSR, the bargaining solution for which will reflect the negotiation outcome: $\bar{q} = q_t$ (see Appendix for proof). For this design, the expected values are equated at:

$$EV_A = EV_B = \int_{\frac{1}{2}(1-\hat{h}_A-\hat{h}_B)}^{\frac{1}{2}(1+2\alpha-\hat{h}_A-\hat{h}_B)} \frac{\hat{h}_A + \hat{h}_B - 1}{2} + \theta_t \ dF_\theta(\theta_t) + \alpha \left[1 - F_\theta\left(\frac{1 + 2\alpha - \hat{h}^A - \hat{h}^B}{2}\right)\right], \tag{33}$$

As described before, the full correlation of benefit factors increases contract value as it rules out change in bargaining positions as a cause for canceling the contract in any period. In this case, the attractiveness of a BSR will depend only on the likelihood of the sum of the benefit factors being higher than the required investments.

3.4 Asymmetric Nash Bargaining

In previous sections, the outcome of the negotiations over the parties' contributions in a single period was based on the assumption that surplus is divided 50:50 (i.e. they maximize an unweighted Nash product). This determines q_t, which is an important element in deriving the various conditions and results discussed so far. If instead, party A takes a share γ of the surplus, while B takes $(1-\gamma)$, the bargaining solution for period t will maximize a weighted Nash product:

$$\underset{q_t}{\text{Max}} \quad (h_t^A - q_t)^\gamma \times (h_t^B + q_t - 1)^{1-\gamma}, \tag{34}$$

as will the bargaining solution for the BSR:

$$\underset{\bar{q}}{\text{Max}} \quad (EV^A)^\gamma \times (EV^B)^{1-\gamma}. \tag{35}$$

From (34), the resulting one-period (re)negotiation outcome is

$$q_t = h_t^A - \gamma(h_t^A + h_t^B - 1). \tag{36}$$

To see how the results change for moderate deviations from $\gamma = \frac{1}{2}$ (i.e. deviations from symmetry), attention is again limited to the range of benefit factors where the public good is always worth the investment and negotiation cost to both parties. This requires a stronger version of condition (9):

$$\frac{h_t^A + h_t^B - 1}{\alpha} > \max\{\frac{1}{\gamma}, \frac{1}{1-\gamma}\}. \tag{37}$$

Having redetermined q_t as a function of the two benefit factors, the values both parties attach to the contract are still given by (22). A BSR that reflects the expected (re)negotiation outcome,

$$\bar{q} = \mu_q = \mu_A - \gamma(\mu_A + \mu_A - 1) = E[q_t], \tag{38}$$

would minimize the probability of renegotiation, thereby maximizing the sum of BSR values. This favors the stronger bargainer, driven by the fact that this party can attach more value to the per-period outside option of negotiations, where it would exploit this strength. However, it is not the bargaining solution for $\gamma \neq \frac{1}{2}$.

Proposition 2. *For moderate deviations from symmetric bargaining strength and assuming (36) holds, the BSR will favor the stronger bargainer beyond the expected advantage when negotiating per period.*

The proof is given in the Appendix. It shows that for the $\bar{q}$ given in (38), the first-order condition for (35) is not satisfied, and its derivative indicates the solution to have lower contributions for whoever has the greater bargaining strength.

Under the circumstance considered here, the advantage from being the stronger bargainer is amplified when the parties negotiate a a BSR. In case there is a threshold BSR value below which it is not worth entering negotiations, such asymmetry in bargaining strength might jeopardize its establishment. Expecting an amplified disadvantage when negotiating the the rule, the weaker bargainer might decide not to enter these negotiations in the first place, e.g. if the expected value is lower than the costs of BSR-negotiations.

It could therefore be beneficial, to both parties, for the stronger party to be able to commit to not exploiting its strength in the contract negotiations. To illustrate this, a simple example with deterministic benefit factors is provided in the Appendix. It is shown that for certain ranges of γ, the asymmetric bargaining strength prevents the establishment of a BSR that would satisfy (35). Both parties would then be better off with a BSR as specified in (38), but this is only a feasible outcome if the stronger bargainer can commit to equalizing BSR-values *before* both parties decide whether or not to enter negotiations.

3.5 Asymmetric α

So far, the cost of negotiation α has been assumed to be the same for both parties. To relax this assumption, denote i's cost of (re)negotiation as α_i. As these costs are sunk once negotiations start, they do not affect the renegotiation outcome:

$$q_t = \frac{1}{2} + \frac{h_t^A - h_t^B}{2}.$$

The range of values for q_t for which the BSR is honored does change, however, for asymmetric α's. We can update this range, as given by (13), to become

$$\bar{q} - \alpha_A \leq q_t \leq \bar{q} + \alpha_B. \tag{39}$$

The expected values of the BSR in one period depends on this range, as well as on the individual negotiation cost:

$$EV_A = \int_{\bar{q}-\alpha_A}^{\bar{q}+\alpha_B} q_t - \bar{q} + \alpha_A \ \ dF_q(q_t),$$

$$EV_B = \int_{\bar{q}-\alpha_A}^{\bar{q}+\alpha_B} \bar{q} - q_t + \alpha_B \ \ dF_q(q_t).$$

Proposition 3. *Under asymmetric negotiations costs, the BSR favors the party with the lower α_i:*

$$\bar{q} = \mu_q + \frac{\alpha_A - \alpha_B}{2}. \tag{40}$$

The proof is given in the Appendix. It is shown that the BSR again minimizes the probability of renegotiation, by equating the density of q_t at the boundaries of the BSR-range. For a symmetric distribution of q_t, this is not achieved by the expected renegotiation outcome μ_q, but requires adjustment to any asymmetry in negotiation costs:

$$f_q(\bar{q} + \alpha_B) = f_q(\bar{q} - \alpha_A), \tag{41}$$

One can interpret the lower contributions for the party with the lower negotiation cost as being caused by two underlying aspects. First, a lower α_i means that party i will need less (upwards) deviation from its expected bargaining position (in $t \geq 1$) to trigger renegotiations. The lower BSR contributions therefore reduce the probability for renegotiation, benefiting both parties. Second, party i enjoys a stronger bargaining position at $t = 0$, as the outside option of not having a BSR is more costly to the other party.

4 Conclusion

To analyze how burden sharing rules may have value and how they are negotiated, I have examined a setting with fairly simplistic features, such as history independent states of nature and a public good that involves no more then two potential contributing parties. Looking at the expected value generated to each party by a BSR has given some insight into what factors affect the outcome of ex ante BSR negotiations and the general attractiveness of default rule, when legally binding contracts are not an option.

The model shows that a non-binding rule can be of value, even when benefits and therefore bargaining positions will change between periods. The cost of renegotiation and the extent to which relative bargaining positions may vary, determine the probability of a BSR being honored, which is a key determinant for its value. While the BSR can increase expected payoffs for both parties, it also increases the payoff variation as it reduces the scope for risk sharing. When parties are risk-averse, this feature reduces its value.

The baseline scenario analysis revealed that the bargaining solution for the BSR sets default contributions equal to the expected per-period negotiation outcome. This BSR design minimizes the probability of renegotiation, and under risk neutrality holds regardless of individual benefit variance.

By relaxing assumptions of symmetry one at the time, the later subsections showed how the BSR design is can deviate from the expected negotiation outcome. Asymmetric benefit risk, for risk-averse preferences, seems to shift the contributions in favor of the party with lower individual risk. A similar result was obtained for asymmetry in negotiation costs, where the party with the lower cost will contribute less under the BSR. Asymmetric bargaining power changes the expected negotiation outcome, but the analysis showed that the BSR may favor the stronger bargainer even further, possibly to such an extent that it the rule becomes infeasible.

Further research could examine the effects of adding one or more parties, or relaxing the assumption of history independent public good benefits. This may affect both the design and the sustainability of a rule, with the latter modification adding the question of when to renegotiate the burden sharing rule itself.

A Appendix

A.1 Proof for Lemma 1

Cooperative contribution levels satisfy

$$\max_{q_t}(h_t^A Q_t - q_t^A) \times (h_t^B Q_t - q_t^B)$$

Replacing q_t^B by $(Q_t - q_t^A)$:

$$\max_{q_t^A, Q_t}(h_t^A Q_t - q_t^A) \times (h_t^B Q_t - Q_t + q_t^A)$$

Differentiating w.r.t. q_t^A gives a first-order condition for q_t^A:

$$h_t^A Q_t - q_t^A = h_t^B Q_t - Q_t + q_t^A$$

$$\iff q_t^A = \frac{(h_t^A - h_t^B)Q_t + Q_t}{2}$$

Gains from cooperation then require

$$h_t^A - \frac{(h_t^A - h_t^B)Q_t + Q_t}{2} = \frac{(h_t^A + h_t^B - 1)Q_t}{2} > 0$$

$$\iff h_t^A + h_t^B > 1.$$

To see that when this condition holds, $Q_t = 1$, express the maximization of the Nash product (NP) as

$$\max_{Q_t}(\frac{(h_t^A + h_t^B - 1)Q_t}{2}) \times (\frac{(h_t^A + h_t^B - 1)Q_t}{2})$$

and note that

$$\frac{\partial NP}{\partial Q_t} = (h_t^A + h_t^B - 1)(h_t^A + h_t^B - 1)Q > 0$$

A.2 Proof of Proposition 1

To see that $\bar{q} = E[q_t] = \mu_q$. satisfies first-order condition

$$\frac{\partial EV^A}{\partial \bar{q}} EV^B + \frac{\partial EV^B}{\partial \bar{q}} EV_A = 0,$$

note that

$$\frac{\partial EV^A}{\partial \bar{q}} EV^B = \left(2\alpha f_q(\bar{q}+\alpha) - \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} f_q(q_t)\ \ dF_q(q_t)\right)\left(\int_{\bar{q}-\alpha}^{\bar{q}+\alpha} \bar{q} - q_t + \alpha\ \ dF_q(q_t)\right)$$

and

$$\frac{\partial EV^B}{\partial \bar{q}} EV^A = -\left(2\alpha f_q(\bar{q}-\alpha) - \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} f_q(q_t)\ \ dF_q(q_t)\right)\left(\int_{\bar{q}-\alpha}^{\bar{q}+\alpha} q_t - \bar{q} + \alpha\ \ dF_q(q_t)\right).$$

At $\bar{q} = E[q_t] = \mu_q$, we have

$$\frac{\partial EV^A}{\partial \mu_q} EV^B = \left(2\alpha f_q(\mu_q+\alpha) - \int_{\mu_q-\alpha}^{\mu_q+\alpha}\ \ dF_q(q_t)\right)\left(\int_{\mu_q-\alpha}^{\mu_q+\alpha} \mu_q - q_t + \alpha\ \ dF_q(q_t)\right)$$

and

$$\frac{\partial EV^B}{\partial \mu_q} EV^A = -\left(2\alpha f_q(\mu_q-\alpha) - \int_{\mu_q-\alpha}^{\mu_q+\alpha}\ \ dF_q(q_t)\right)\left(\int_{\mu_q-\alpha}^{\mu_q+\alpha} q_t - \mu_q + \alpha\ \ dF_q(q_t)\right).$$

For a single-peaked distribution F_q that is symmetric around its mean μ_q, the first-order condition is satisfied.

A.3 Proof for one-sided risk

For the expected values we can rewrite the limits of the BSR-range as expressions of only h^A, to obtain

$$EV^A = \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u(h_t^A - \bar{q}) - u(h_t^A - \frac{h_t^A - \hat{h}^B + 1}{2} - \alpha)\ dF_A(h^A),$$

$$EV^B = \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u(\hat{h}^B - 1 + \bar{q}) - u(\hat{h}^B - 1 + \frac{h_t^A - \hat{h}^B + 1}{2} - \alpha)\ dF_A(h^A).$$

Therefore,

$$\frac{\partial EV_A}{\partial \bar{q}} = 2f_A(\hat{h}^B+2\bar{q}-1+2\alpha)\left[u(\hat{h}^B + \bar{q} - 1 + 2\alpha) - u(\hat{h}^B + \bar{q} - 1)\right] - \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u'(h_t^a - \bar{q})\ dF_A(h^A)$$

$$\frac{\partial EV_B}{\partial \bar{q}} = -2f_A(\hat{h}^B+2\bar{q}-1-2\alpha)\left[u(\hat{h}^B + \bar{q} - 1) - u(\hat{h}^B + \bar{q} - 1 - 2\alpha)\right] + \int_{\hat{h}^B+2\bar{q}-1-2\alpha}^{\hat{h}^B+2\bar{q}-1+2\alpha} u'(\hat{h}^B + \bar{q} - 1)\ dF_A(h^A)$$

At $\bar{q} = \frac{\mu_A - \hat{h}_B + 1}{2} = E[q_t]$, the above four equations become

$$EV^A = \int_{\mu_A-2\alpha}^{\mu_A+2\alpha} u(h_t^A - \frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2}) - u(\frac{h_t^A + \hat{h}^B - 1}{2} - \alpha)\ dF_A(h^A),$$

$$EV^B = \int_{\mu_A-2\alpha}^{\mu_A+2\alpha} u(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2}) - u(\frac{h_t^A + \hat{h}^B - 1}{2} - \alpha)\ dF_A(h^A).$$

And,

$$\frac{\partial EV_A}{\partial \bar{q}} = 2f_A(\mu_A+2\alpha)\left[u(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2} + 2\alpha) - u(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2})\right] - \int_{\mu_A-2\alpha}^{\mu_A+2\alpha} u'(h_t^A - \frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2}) dF_A(h^A)$$

$$\frac{\partial EV_B}{\partial \bar{q}} = -2f_A(\mu_A-2\alpha)\left[u(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2}) - u(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2} - 2\alpha)\right] + \int_{\mu_A-2\alpha}^{\mu_A+2\alpha} u'(\frac{\mu_A}{2} + \frac{\hat{h}_B - 1}{2}) dF_A(h^A)$$

Given the assumptions on $u()$,

$$EV_A < EV_B$$

$$\frac{\partial EV_A}{\partial \bar{q}} < 0, \qquad \frac{\partial EV_B}{\partial \bar{q}} > 0 \tag{42}$$

$$\frac{\partial EV_A}{\partial \bar{q}} + \frac{\partial EV_B}{\partial \bar{q}} < 0, \tag{43}$$

and

$$\frac{\partial EV_A}{\partial \bar{q}} EV_B + \frac{\partial EV_B}{\partial \bar{q}} EV_A < 0. \tag{44}$$

This means that at $\bar{q} = E[q_t]$ $\bar{q}$ is above the value that satisfies the condition for the Nash bargaining solution, which thus requires the BSR-contribution by A to be reduced.

A.4 Bargaining solution section 3.3

For fully correlated risk and an unrestricted range for the common shock, we have

$$EV^A = \int_{\max\{\bar{q}-\hat{h}^A, 1-\bar{q}-\hat{h}^B\}}^{\frac{1}{2}(1+2\alpha-\hat{h}^A-\hat{h}^B)} \hat{h}^A + \theta_t - \bar{q} \;\; dF_\theta(\theta_t) + \int_{\frac{1}{2}(1+2\alpha-\hat{h}^A-\hat{h}^B)}^{\infty} q_t - \bar{q} + \alpha \;\; dF_\theta(\theta_t), \tag{45}$$

$$EV^B = \int_{\max\{\bar{q}-\hat{h}^A, 1-\bar{q}-\hat{h}^B\}}^{\frac{1}{2}(1+2\alpha-\hat{h}^A-\hat{h}^B)} \hat{h}^B - 1 + \bar{q} \;\; dF_\theta(\theta_t) + \int_{\frac{1}{2}(1+2\alpha-\hat{h}^A-\hat{h}^B)}^{\infty} \bar{q} - q_t + \alpha \;\; dF_\theta(\theta_t). \tag{46}$$

Note that for any BSR that generates any value,

$$\max\{\bar{q} - \hat{h}^A, 1 - \bar{q} - \hat{h}^B\} \quad \leq \quad \frac{1}{2}(1 + 2\alpha - \hat{h}^A - \hat{h}^B).$$

This becomes clearer when we rewrite the above condition te become

$$\bar{q} - \alpha \leq q_t \leq \bar{q} + \alpha,$$

where q_t is as in (31). Now, if this does not hold, the BSR would *never* be effectuated:

1. For any $h_t^A + h_t^B \geq 1 + 2\alpha$, the BSR would be renegotiated $\rightarrow$ no value.
2. If $h_t^A + h_t^B < 1 + 2\alpha$:
   - The sum of the payoffs is at most 2α: $\quad \bar{\pi}^A + \bar{\pi}^B \leq 2\alpha$
   - The difference between the BSR-payoffs will be larger than that: $|\bar{\pi}^A - \bar{\pi}^B| \geq 2\alpha$
   - It follows that at least one of the two parties will have negative payoff under the BSR, and would opt out $\rightarrow$ no value.

To show that the the Nash product of the two expected BSR values is maximized at $\bar{q} = q_t = \frac{\hat{h}^A - \hat{h}^B + 1}{2}$, note that at this $\bar{q}$ we have $EV^A = EV^B$. What remains to be shown is that at the same time $\frac{\partial EV^A}{\partial \bar{q}} = -\frac{\partial EV^B}{\partial \bar{q}}$.

For the derivatives we have to look at two ranges for $\bar{q}$. First consider $\bar{q} \geq q_t \iff \bar{q} - \hat{h}^A \geq 1 - \bar{q} - \hat{h}^B$. Then

$$\frac{\partial EV^A}{\partial \bar{q}} = -\left[1 - F_\theta\left(\bar{q} - \hat{h}^A\right)\right] \quad \text{vs.} \quad \frac{\partial EV^B}{\partial \bar{q}} = -(2\bar{q} - 2q_t) f_\theta(\bar{q} - \hat{h}^A) + \left[1 - F_\theta\left(\bar{q} - \hat{h}^A\right)\right].$$

In the other range, $\bar{q} \geq q_t$, we have

$$\frac{\partial EV^A}{\partial \bar{q}} = -(2\bar{q}-2q_t)f_\theta(1-\bar{q}-\hat{h}^B)-\left[1 - F_\theta\left(1 - \bar{q} - \hat{h}^B\right)\right] \quad \text{vs.} \quad \frac{\partial EV^B}{\partial \bar{q}} = \left[1 - F_\theta\left(1 - \bar{q} - \hat{h}^B\right)\right].$$

From the above equation it is straightforward to see that the first-order condition for the Nash bargaining solution is satisfied at $\bar{q} = q_t$, where $\bar{q} - \hat{h}^A = 1 - \bar{q} - \hat{h}^B$.

A.5 Proof of Proposition 2

If we abbreviate the weighted Nash product as

$$NP(\gamma, \bar{q}) = \quad (EV^A(\bar{q}))^\gamma \times (EV^B(\bar{q}))^{1-\gamma}, \tag{47}$$

Proposition 2 requires the following to be true:

$$\frac{\partial NP\left(\gamma, E\left[q_t\right]\right)}{\partial \bar{q}} < (>)0 \quad \text{if} \quad \gamma > (<)\frac{1}{2},$$

where

$$\frac{\partial NP(\gamma, \bar{q})}{\partial \bar{q}} = \gamma\left(\frac{EV^B}{EV^A}\right)^{1-\gamma}\frac{\partial EV^A}{\partial \bar{q}} + (1-\gamma)\left(\frac{EV^A}{EV^B}\right)^{\gamma}\frac{\partial EV^B}{\partial \bar{q}} \tag{48}$$

Similar to the proof for Proposition 1, it is useful to define a distribution F_q, that is now constructed from the two benefit factors according to

$$q_t = h_t^A - \gamma(h_t^A + h_t^B - 1). \tag{49}$$

Relative to this q_t, the BSR-values are determined by $\bar{q}$ in the same way as for symmetric bargaining strength:

$$EV^A = \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} q_t - \bar{q} + \alpha \;\; dF_q(q_t), \tag{50}$$

$$EV^B = \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} \bar{q} - q_t + \alpha \;\; dF_q(q_t).$$

For $\bar{q} = E[q_t]$, and distributions of the random variables that are symmetric around their means, these values are equalized: $EV^A = EV^B$. This reduces (48) to

$$\frac{\partial NP}{\partial \bar{q}} = \gamma\frac{\partial EV^A}{\partial \bar{q}} + (1-\gamma)\frac{\partial EV^B}{\partial \bar{q}} \tag{51}$$

For the partial derivatives we (again) obtain

$$\frac{\partial EV^A}{\partial \bar{q}} = 2\alpha f_q(\bar{q}+\alpha) - \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} \;\; dF_q(q_t)$$

and

$$\frac{\partial EV^B}{\partial \bar{q}} = -2\alpha f_q(\bar{q}-\alpha) + \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} dF_q(q_t).$$

For a symmetric, single peaked distribution F_q, densities for q-values inside the range $[\mu_q - \alpha, \mu_q + \alpha]$ are greater than outside. Hence, at $\bar{q} = \mu_q$ we have $\frac{\partial EV^A}{\partial \bar{q}} < 0$ and $\frac{\partial EV^A}{\partial \bar{q}} > 0$, while the sum of the two is equal to zero. This is therefore only the bargaining solution if $\gamma = \frac{1}{2}$.

If not, it follows from (51) that the sign of the derivative matches that of $(1-2\gamma)$.

A.6 Example asymmetric bargaining strength

To illustrate how asymmetric bargaining strength might prevent a potentially beneficial BSR, consider the following simple example:

- Benefit factors are deterministic, assume: $h^A = h^B = \frac{3}{4}$.
- A is assumed to be the stronger bargainer: $\gamma > \frac{1}{2}$.
- Benefits are large enough for B to enter negotiations in $t \geq 1$: assume $\alpha < \frac{1-\gamma}{2}$
- $T = 2 \quad \rightarrow \quad$ There are two periods of possible public good provision, after $t = 0$ where the parties may or may not establish a BSR
- Assume that the parties do not discount future payoffs, and that there is cost α^0 to negotiating the BSR at $t = 0$ that is equal to the cost for per-period negotiations: $\alpha^0 = \alpha$.

By (36), we obtain the (re)negotiation outcome in periods 1 and 2:

$$q_t = \frac{3-2\gamma}{4}.$$

Without a BSR, the surplus subject to negotiations in both periods 1 and 2 is certainly $\frac{1}{2}$. Taking into account that these negotiations are costly, we know that party B will end up with a payoff of $\pi_t^B = \frac{(1-\gamma)}{2} - \alpha$. The BSR-values in each period $t \geq 1$ are given by

$$V^A = \frac{3-2\gamma}{4} - \bar{q} + \alpha \quad \text{and} \quad V^B = \bar{q} - \frac{3-2\gamma}{4} + \alpha.$$

Note that, as before, if the BSR is set at $\bar{q} = E[q_t] = q_t$, the BSR-value in period t will be equal: $V^A = V^B = \alpha$. The bargaining solution for $\bar{q}$ will be different, however, for unequal bargaining strength:

$$\max_{\bar{q}} \quad (2V^A)^\gamma \times (2V^B)^{1-\gamma}, \tag{52}$$

which gives first-order condition

$$\gamma \left(\frac{V^B}{V^A}\right)^{1-\gamma} = (1-\gamma)\left(\frac{V^A}{V^B}\right)^{\gamma}. \tag{53}$$

Using that $V^A + V^B = 2\alpha$, we obtain

$$V^A = 2\gamma\alpha \quad \text{and} \quad V^B = (1-\gamma)2\alpha. \tag{54}$$

Now assume that A is the stronger bargainer, i.e. $\gamma > \frac{1}{2}$. Given the cost $\alpha^0 = \alpha$ associated with the BSR-negotiations, B will only be willing to enter those negotiations if

$$2V^B \geq \alpha \qquad \Longleftrightarrow \qquad (1-\gamma)4\alpha \geq \alpha.$$

It is straightforward to see, then, that there will not be a BSR for $\gamma > \frac{3}{4}$. It follows that party A would then be better off if able to commit to a BSR outcome more favorable to B (obviously B would also be better off). To illustrate this, the following table compares A's payoffs without BSR to those with BSR $\bar{q} = q_t$.

Period	**No BSR:**	**BSR $\bar{q} = q_t$:**
t	π^A	$\bar{\pi}^A$
0	0	-α
1	$\gamma/2 - \alpha$	$\gamma/2$
2	$\gamma/2 - \alpha$	$\gamma/2$
Total	$\boldsymbol{\gamma - 2\alpha}$	$\boldsymbol{\gamma - \alpha}$

A.7 Proof of proposition 3

To see that at $\bar{q} = \mu_q + \frac{\alpha_A - \alpha_B}{2}$, the following condition is satsified:

$$\frac{\partial EV^A}{\partial \bar{q}} EV^B = -\frac{\partial EV^B}{\partial \bar{q}} EV^A. \tag{55}$$

First note that for this BSR, under the same assumptions on F_q as before,

$$EV_A = EV_B = \frac{\alpha_A + \alpha_B}{2} \int_{\bar{q}-\alpha_A}^{\bar{q}+\alpha_B} dF_q(q_t).$$

Second, for the partial derivatives we obtain

$$\frac{\partial EV^A}{\partial \bar{q}} = (\alpha_A + \alpha_B) f_q(\bar{q} + \alpha_B) - \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} dF_q(q_t)$$

and

$$\frac{\partial EV^B}{\partial \bar{q}} = -(\alpha_A + \alpha_B) f_q(\bar{q} - \alpha_A) + \int_{\bar{q}-\alpha}^{\bar{q}+\alpha} dF_q(q_t).$$

Condition (56) therefore requires

$$f_q(\bar{q} + \alpha_B) = f_q(\bar{q} - \alpha_A), \tag{56}$$

Plugging in the proposed $\bar{q}$ makes it clear that (57) holds:

$$f_q(\mu_q + \frac{\alpha_A - \alpha_B}{2} + \alpha_B) = f_q(\mu_q + \frac{\alpha_A - \alpha_B}{2} - \alpha_A).$$

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