The Political Economy of Multilateral Aid Funds*

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Abstract

When allocating foreign aid, donor countries face a problem of incentivizing recipient countries to invest in state capacity. In particular, a "Samaritan's" problem occurs when donors cannot commit to tie aid to the level of recipient-country investment. While delegating aid decisions to a third party has been suggested as a solution to the Samaritan's problem, this solution requires the existence of an independent party with specific preferences that, say, prioritize efficiency over need. We show that, alternatively, donors can mitigate the Samaritan's problem by committing to a multilateral fund: if aid allocation decisions are made ex post via bargaining between donors, then the negotiated outcome will be skewed towards aggregate efficiency, which induces the recipients to compete over ex ante investments. Our model links the fund's composition of membership and decision rules to participation, investment and allocation decisions. We show that delegation to a multilateral fund is optimal between donors that place similar utility weights on aid spending, but diverge in their preferences over which recipients should receive funds. Also, we find that while majority rule induces stronger competition between recipients, it limits aid to a strict subset of recipient countries, which implies that unanimity is often the optimal decision rule.

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When allocating foreign aid, donor countries face a problem of incentivizing recipient countries to invest in state capacity in an effort increase the effectiveness of aid spending. To address this problem, a new wave of aid conditionality - political conditionality - has emerged with the intention of incentivizing needed investments and reforms (see Molenaers et al. (2015)). However, political conditionality suffers from the same difficulty with credible implementation that has lead to many experts to claim that aid conditionality has failed (see Collier, 1997, Alesina and Weder, 2002, Dreher, 2009 and Öhler et al., 2012). In particular, conditional aid faces a problem of non-contractibility – measures of good governance are partially subjective, and donor countries often face an ex post incentive to circumvent conditionality (for example, Stone, 2004 documents that countries with strong ties to the US are less likely to face sanctions for violating IMF conditionality). The problem of non-contractibility results in a "Samaritan's problem" that occurs when the recipient country, knowing it will receive assistance in any case, has no incentive to implement costly reforms (see Mosley et al., 1995 and Pedersen, 1996 for a discussion of problems of time-inconsistency in aid spending).

Given the difficulty of implementing conditionality in bilateral aid, several papers consider the delegation of the allocation decision to the third party as a solution to the Samaritan's problem (see Svensson (2000), Hagen (2006) and Annen and Knack (2015)). Delegation is a well-known solution to hold-up problems in general (see Aghion and Tirole, 1997), and may allow commitment to allocate based on recipient-country investment in the particular case of the Samaritan's problem. However, this solution depends on the existence of an independent party with verifiable preferences that diverge from the donor country's in the precise direction that facilitates recipient-country investment. Therefore, delegation to international aid agencies may not always mitigate the Samaritan's problem – as argued by Easterly (2003) and Hagen (2006), aid agencies often focus on ease of disbursement or recipient-country need rather than aid effectiveness.

In this paper, we consider an third option: commitment to collective decision-making. That is, instead of delegating to an independent party, donor nations may commit aid to a jointly-managed (multilateral) fund – effectively delegating control of aid allocation decisions to a bargaining process between donor-country representatives. We show that delegating aid to a multilateral fund mitigates the Samaritan's problem, even in the extreme case where donor countries individually have no ex post incentive to allocate aid based on efficiency. Moreover, this solution to the Samaritan's problem does not rely on delegation to an institution with an exogenous objective – instead, the objective of the fund emerges endogenously from the bargaining process and the preferences of the individual donor countries.

While the Samaritan's problem is a general problem in foreign aid, it is particularly relevant for aid directed at providing global public goods, such as climate change aid, where efficiency rather than need dictates optimal aid allocation (Bagchi et al. (2016)). This sector is both large and growing: in 2014, donor countries distributed over $60 billion in climate change aid – roughly equivalent to the total amount of development assistance provided by the World Bank for the same year – and the Paris Agreement set a goal of mobilizing $100 billion per year by 2020 (OECD, 2015, World Bank, 2015). Rather than being centralized under one institution, however, climate change aid is administered by a variety of different funds with different institutional rules: currently, at least nine multilateral funds have been established, such as the Green Climate Fund (pledged budget: $10.2 billion) and the Least Developed Countries Fund ($964 million), as well as several bilateral aid funds, such as the UK's International Climate Fund ($6 billion; Nakhooda et al., 2015). Additionally, these funds differ in their formal setup: while some climate aid funds, such as the Least Developed Countries Fund, take allocation decisions via a unanimity rule among country representatives, others, such as the Adaptation Fund, use a two-third majority rule (Climate Funds Update, 2015).

Our model provides an explanation for this dispersion of climate aid across institutions and political setups.[1] That is, by explicitly modeling the bargaining process between donor-county representatives, we are able to characterize the objective of the multilateral fund as an endogenous function of the fund's composition, as well as its formal rules for decision-making. This allows us to address questions such as under which conditions donor countries will form a fund, and which decision rules are optimal for the fund.

We consider a setting of two donor and two recipient countries. The donor countries each have an aid budget, and first choose whether to implement aid bilaterally or via a multilateral fund.[2] The recipient countries then choose an observable, but non-contractible, level of investment that increases the effectiveness of aid spending, after which the donor countries determine the allocation of aid spending.

Importantly, donor countries are biased over which country receives aid – to make the Samaritan's problem as stark as possible, each donor country only values the product of aid spending in one of the recipient countries (for a discussion of bias in bilateral aid spending, see Dreher, 2009). In this setting, the donor countries would like to incentivize the recipient countries to invest ex ante to increase the productivity of aid spending. However, knowing that the donor countries will allocate all spending to their preferred

[1]It is important to note that all aid modalities have access to benefits of scale in implementation since bilateral aid is often implemented by multilateral institutions (see Eichenauer and Hug, 2014 for an analysis of the implementation of earmarked aid).

[2]We focus on contrasting between bilateral aid and multilateral aid funds, and do not consider delegation to an independent third party. The effectiveness of delegation to an independent party in overcoming the Samaritan's problem depends crucially on the objective of that institution (see Svensson (2000)) – here we consider the objective that endogenously emerges from direct bargaining between country representatives.

recipient country ex post under bilateral aid, the recipient countries have no incentive to invest ex ante.

Alternatively, if donor countries commit to allocate via a multilateral aid fund, the donor countries bargain over allocation outcomes ex post à la Nash. And since Nash bargaining results in an allocation outcome that reflects aggregate efficiency (that is, efficiency according to the donor countries' preferences), a recipient country with a higher effectiveness will receive a higher level of aid. Therefore, the commitment to allocate aid via the multilateral fund induces competition over ex ante investments by recipient countries, which in turn mitigates the Samaritan's problem.[3] That is, it is precisely the process of preference aggregation in the multilateral fund that enables multilateral organizations to better implement aid conditionality – when donor countries with heterogenous preferences pool their resources, competition among recipient countries over aid intensifies since bargaining functions as a commitment to reward recipient countries for higher investments.

Despite this strategic advantage of multilateral funds over bilateral aid, our model shows that a donor's decision of whether to join the fund is not always straight-forward. Delegation to a fund effectively requires that the donor countries surrender control over their aid budget to a bargaining outcome, which may expose them to "expropriation" by other donor countries. That is, while delegation to a fund increases recipient-country investment, and hence increases aggregate donor-country utility, Nash bargaining also implements an equal division of the utility surplus from aid spending. Therefore, donor countries with relatively high utility-weights on the product of aid spending face a tradeoff when deciding whether to allocate aid multilaterally and, intuitively, we show that donor countries will only form a multilateral fund if the relative difference in the countries' utility weights is low enough.[4]

Additionally, by explicitly modeling the bargaining process of the multilateral fund, we are able to link the investment outcomes to the decision rule used to allocate funding. In our baseline analysis we consider a multilateral aid fund that allocates funding via unanimity. In an extension of the model, however, we demonstrate that the recipient countries' incentive to invest in reform is also a function of the decision rule used within

[3] Svensson (2003) details how competition over aid spending incentivizes investment in state capacity. In out setting, donor countries cannot induce such a competition under bilateral aid due to the non-contractibility of investments.

[4] In a paper developed concurrently to ours, Annen and Knack (2015) consider donor countries' decision to allocate aid bilaterally or via an independent third party. A crucial difference is that their model assumes that the multilateral agency will implement the outcome that maximizes the average payoff of the donor countries. Therefore, they find that donor countries will allocate multilaterally only when donor country preferences over the recipient countries are sufficiently homogenous. In contrast, our approach explicitly models the bargaining process between donor-county representatives and finds that donor countries will join the fund even when their preferences over the recipient countries diverge perfectly, as long as the difference in utility weights is low enough. However, the two approaches are difficult to distinguish empirically given the lack of data on intensity of preferences.

the fund. Analogous to Harstad (2005), we show that majority rule further increases the incentive for the recipient countries to invest, since higher investment increases the probability that a recipient's project is selected by the endogenous majority coalition. However, the higher incentive to invest comes at a cost of limiting the total number of projects that are funded, which implies that a majority rule will only outperform unanimity when the utility benefit of investment to the donor countries is relatively low.

Our paper's main contribution is to the literature on aid conditionality (see Dreher, 2009 and Molenaers et al., 2015 for an overview), showing that delegation to a multilateral fund, rather than an independent third party, functions as a commitment to implement conditionality due to the endogenous objective that arises when donor countries bargain over aid allocation. However, our work also contributes to the literature on optimal decision rules in international organizations (see Harstad (2005), Maggi and Morelli (2006), and Barbera and Jackson (2006)). In particular, our results regarding the optimal voting rule in multilateral funds are closely related to Harstad (2005), who shows how a majority rule can mitigate a holdup problem in a setting where investments by members of a club (or countries in a union) are expropriated ex post. A key difference in our findings, however, is that majority rule is not always optimal for donor countries, even though the costs of investments are fully borne by the recipient countries. Instead of simply choosing the decision rule that maximizes investments, unanimity rule may be optimal in our setting because it ensures that all recipient countries projects receive some level of funding, whereas majority rule increases investment precisely by limiting funding to a strict subset of projects.

Our paper proceeds as follows: Section 1 presents the baseline model, and the analysis is contained in section 2. Section 3 goes on to characterize the optimal voting rule within the multilateral aid fund, and section 4 concludes. Formal proofs for all results are presented in the appendix.

1 Model

There are two donor countries and two recipient countries, denoted as $i = 1, 2$ and $j = 1, 2$ respectively.[5] Each donor country has an individual budget for foreign aid $x_i = x$ to allocate across a set of recipient country aid projects, $\{g_1, g_2\}$.

Preferences and Actions

Donor countries initially choose whether to commit their budget to a multilateral aid fund ($f_i = 1$) or to retain control over the allocation of their budget ($f_i = 0$). If one

[5] For simplicity, we consider a baseline model of only two countries – while the main insights of the model remain unchanged with more countries, the comparative statics are difficult to characterize with $n > 2$.

of the donor countries chooses bilateral aid ($f_i = 0$) then, in the last stage of the game, each country simultaneously chooses how to allocate x over $\{g_1, g_2\}$. However, if both donor countries choose $f_i = 1$, then the multilateral aid fund allocates the full budget, $2x$, over $\{g_1, g_2\}$ via Nash Bargaining (NB) between the donor countries. In the bargaining process, each country has an equal bargaining weight, and the fund has access to utility transfers.[6]

Each donor country assigns different utility-weights on aid spending across the recipient countries, represented by the vector $\{\beta^i_j\}$. These weights could for example reflect preferences due to geographical proximity, trade relations or historical ties. In addition to the donor country weights, each recipient country has a "quality-weight," of $\alpha_j = 1 + \delta_j$. One may for example think about governance structures that impact the effectiveness of aid. Each recipient country can influence its own quality weight by investing in $\delta_j \geq 0$ according to the cost function $c(\delta_j) = (\delta_j + 1)^2 - 1$ or, equivalently, $c(\alpha_j) = (\alpha_j)^2 - 1$. This structure can be interpreted as each project having a minimum quality level of $\alpha_j = 1$, above which active investment by the recipient countries is required to improve project quality. Investment, however, is observable but non-contractible, which implies only a limited scope for donor countries to condition allocations on investments.

Donor countries have preferences over the set $\{g_j\}$ that are increasing and concave. Specifically, we assume

$$u_i(\{g_j, \alpha_j, \beta^j_i\}) = \sum_j \alpha_j \beta^j_i \sqrt{g_j} + \lambda^j_i \delta_j.$$

Note that while donor countries do not directly take into account the utility recipient countries receive from their aid spending, we do allow for the possibility that donor countries may value δ_j independently from its impact on the quality of the project, reflected in the term $\lambda^j_i \delta_j$ (we refer to this as the indirect benefit of investment). The main insights of our model do not rely upon this feature and, unless explicitly stated, all results go through with $\lambda^j_i = 0 \; \forall \; i, j$. However, given that good governance in recipient countries is a common policy goal of donor countries (as discussed in the introduction), and since recipient country investments may have spillover effects to other policy areas, we find it relevant to account for donor countries' preferences for, say, decreased corruption.

Recipient countries have linear preferences over aid spending g_j:

$$u_j(g_j, \delta_j) = g_j - ((\delta_j + 1)^2 - 1) \equiv g_j - ((\alpha_j)^2 - 1).$$

[6]Utility transfers ensure that donor countries split the utility surplus equally – without utility transfers, the NB results in an outcome that balances efficiency (aggregate utility) and equity (see Simon and Valasek, 2016 for more detail). Therefore, our main results remain qualitatively similar even without utility transfers.

Note that recipient countries only value the direct spending in their own country whereas donor countries may also value spillovers from projects they do not fund themselves. For this analysis, however, we assume that $\beta_i^j = 0$ for $i \neq j$ and $\beta_i^i = \beta_i > 1$, and that $\lambda_i^j = 0$ for $i \neq j$ and $\lambda_i^i = \lambda$. This introduces stark heterogeneity between donor countries in terms of their preferences across recipient countries (each donor country cares only about one specific recipient country) and allows for differences in valuations of aid in general (if $\beta_i \neq \beta_k$). These restrictions allows us to clearly illustrate the main points of the model. With the simplifying assumptions, each donor nation's utility function becomes:

$$u_i(\alpha_i, g_i) = \alpha_i \beta_i \sqrt{g_i} + \lambda \delta_i. \tag{1}$$

Timing and Equilibrium:

The timing of the game is as follows:

1. Donor countries choose whether to allocate aid bilaterally or join their resources in an aid fund (f_i).
2. Recipient countries choose investment levels, δ_j (α_j).
3. Aid is allocated either bilaterally - i.e. each donor country chooses $g_i \leq x_i$ - or centrally - i.e. donor countries bargain á la Nash over $\{g_i\}$ with $\sum g_i \leq \sum x_i$.

The equilibrium we utilize is analogous to sub-game perfect Nash equilibrium, with the exception that, if the donor countries choose to join their resources in an aid fund, the allocation decision is determined via NB.

Throughout the analysis, we consider the objective of the donor countries, rather than, say, an objective of maximizing aggregate utility. We argue that this is a natural objective to consider when analyzing the political economy of multilateral aid funds. However, this does not imply that investments carried out by the recipient countries' should be considered as non-productive for the *population* of these countries: while we assume that investment in good governance is costly for the *regime* of the recipient country, it is possible that these reforms provide utility benefits to the recipient countries' population by increasing the effectiveness of the public sector.

2 Analysis

We solve the model by backward induction, and thus begin with the allocation decisions.

2.1 Allocation of aid

In the last step, donor countries decide how to allocate the aid budget to the set of projects $\{g_1, g_2\}$. They may decide not to spend (all of) the budget, but cannot at this point reverse their decision on whether to allocate bilaterally or jointly through a fund.

Bilateral allocation

If countries have chosen to bilaterally allocate their aid budget, each donor country i solves

$$\max_{\{g_1,g_2\}} \quad \alpha_i \beta_i \sqrt{g_i} \tag{2}$$

$$\text{s.t.} \quad g_1 + g_2 \leq x \tag{3}$$

Lemma 1
Irrespective of the level of α_i, each donor country i will spend all of its budget in recipient country i, i.e. $g_i = x$.

This result already eludes to the samaritan's dilemma donor countries face when allocating their aid budget bilaterally: Donors will always spend all their aid money, *irrespective* of the level of reform effort their target nation decides to implement. Even though they are free to reduce aid spending, their preferences make it impossible to effectively commit to such conditionality.

Joint fund

If donors have decided to allocate their aid budgets through a joint fund, at this stage they bargain over the allocation of the aggregate budget to the set of recipient country projects. Since utility transfers are possible, the Nash bargaining outcome maximizes the sum of utilities of the bargaining parties:[7]

$$\max_{\{g_i\}_{i=1}^n} \quad \sum_i \alpha_i \beta_i \sqrt{g_i} \tag{4}$$

$$\text{s.t.} \quad g_1 + g_2 \leq 2x \tag{5}$$

Note that the indirect utility benefit of recipient-country investment is not factored into the NB outcome, since $\lambda\delta_i$ is independent of the allocation decision (i.e. the indirect utility benefit of investment is part of each donor country's outside option).

The NB outcome implies the following lemma:

[7] A complete description of the NB outcome would include utility transfers; however, since there is no need to refer to them directly, we simplify the notation by not explicitly introducing the utility transfers.

Lemma 2

The fund will also spend the complete budget irrespective of the levels of α_j, i.e. $g_1+g_2 = 2x$. The division of funds, however, does depend on the levels of reform investments:

$$g_i = \frac{(\alpha_i\beta_i)^2}{(\alpha_1\beta_1)^2 + (\alpha_2\beta_2)^2} 2x \tag{6}$$

Unlike with bilateral aid allocation, the share of the total aid budget that each target nation receives is sensitive to its investment in α_j.

Since utility transfers are possible, donor countries share the created surplus equally, i.e. they each receive:

$$u_i = \frac{1}{2} \sum_i \alpha_i \beta_i \sqrt{g_i}. \tag{7}$$

2.2 Investment decisions

Recipient countries move simultaneously when deciding their investment levels α_j (for convenience, we refer to recipient countries setting α_j, rather than δ_j, subject to the constraint that $\alpha_j \geq 1$). They also know the mechanism by which aid will be allocated (bilaterally or through a fund) and thus take into account how their share of aid will change with the investment they make.

Each recipient country chooses α_j to solve:

$$\max_{\alpha_j} \left\{ g_j(\{\alpha_k\}_{k=1}^n) - (\alpha_j^2 - 1) \right\}. \tag{8}$$

Bilateral aid

When aid is allocated bilaterally, recipient countries know that the donor who prefers to allocate aid to their project will do so irrespective of investments in α. That is, recipient countries solve problem 8 with $g_j = x$.

Lemma 3

When aid is allocated bilaterally, recipient countries do not invest in reforms beyond the minimum level, i.e. $\alpha_j = 1$ for all j.

This is a classic hold-up problem (aid allocation does not increase to reflect the increased investment). A minimum level of α is guaranteed to be implemented. Beyond that, donor countries face significant difficulties to actually implement aid conditionality when allocating aid bilaterally.

Multilateral aid fund

Here the recipient countries internalize the effect their reform efforts have on the final allocation of the fund's budget. That is, each recipient country choses α_j to maximize their utility, taking as given the allocation rule (6) of the donor countries and the investment decisions of the other recipient countries. This optimization problem yields

$$\alpha_j = \max\{1; \sqrt{2x}\frac{\beta_j \beta_i}{\beta_1^2 + \beta_2^2}\} \quad \forall j \tag{9}$$

Proposition 1
Reform investment is always (weakly) greater when aid is allocated through a fund.

Proposition 1 details the main insight of the analysis: When aid is allocated through a joint fund, the bargaining process induces competition between the recipient countries. They thus have an incentive to invest in reforms in order to secure a larger share of the budget. Such competition cannot be induced through bilateral aid when donor countries are biased over the allocation of aid.

It is interesting to consider some comparative statics of the reform effort of recipient countries.

Corollary 1
Reform effort is increasing in the fund's budget.

This result is straightforward: The larger the pie recipient countries are now directly competing over, the higher the incentive to invest in reforms that will secure a larger part of the total aid budget.

Corollary 2
Reform effort is (weakly) decreasing in asymmetry between the donor countries' valuations β_i.

To see this, it helps to rewrite expression 9 in terms of the ratio $B = \frac{\beta_1}{\beta_2}$:

$$\alpha_j = \sqrt{2x}\frac{B}{1 + B^2} \tag{10}$$

α_j is maximized when $B = 1$ and decreases as one moves away from $\beta_1 = \beta_2$ in either direction. Thus, a recipient country's incentives to invest in order to increase its aid share are higher the more equal the exogenously given donor valuations β are. Once donor countries are asymmetric in their own valuations, recipient country 1's investment incentives decrease - either because it is expensive to "catch up" with the other country's higher ex-ante valuation (in case $\beta_1 < \beta_2$), or because it is unnecessary to invest more, since country 2 finds it too expensive to catch up (in case $\beta_1 > \beta_2$).

2.3 Equilibrium Decision: Which donor prefers the fund?

Lastly, we consider the choice of donor countries of whether join an aid fund or to allocate its aid budget bilaterally: The optimal decision comes from comparing payoffs under each scenario, based on the anticipation of the donor countries regarding how the fund will influence the investment decisions of the recipient countries. Donor country i will find it optimal to join the fund ($f_i = 1$) when:

$$F_i = \frac{1}{2}[\alpha_i \beta_i \sqrt{g_i} + \alpha_j \beta_j \sqrt{g_j}] + \lambda \delta_i - \beta_i \sqrt{x} \tag{11}$$

$$= \frac{1}{2}\alpha\sqrt{2x}\frac{\beta_i^2 + \beta_j^2}{\sqrt{\beta_i^2 + \beta_j^2}} + \lambda\delta - \beta_i\sqrt{x} \geq 0. \tag{12}$$

This comparison involves the following trade-off: Through joining the fund, the donor country increases the level of investment, δ_i, chosen by the recipient country, but at the same time commits to equally share the utility surplus from aid spending among the donors. Both of these considerations depend on the level of asymmetry in βs as well as the size of the overall budget.

Proposition 2
When donor countries are symmetric ($\beta_1 = \beta_2$), it is optimal for both donor countries to commit their aid budgets to an multilateral aid fund ($f_i = 1$ for $i = 1, 2$).

With symmetric βs, the equilibrium allocation of funds is exactly equal to the allocation under bilateral aid (i.e. each project receives x). Thus, there is only the upside of increased investments in α, which makes the fund always (weakly) more attractive than bilateral aid.

Proposition 3
When donor countries are not symmetric, holding β_j constant, there exists a cut-off level of β_i, β', such that for $\beta_i > \beta'$, donor country i sets $f_i = 0$ (i prefers bilateral aid over joining the fund).

Proposition 3 illustrates that when the donor countries biases are asymmetric, then the decision to commit to allocate aid via the fund is costly to the donor country with the greater bias. If this asymmetry is high enough, then the country with the higher bias will prefer to allocate aid bilaterally. Note, however, that this result only depends on the *relative* size, rather than the absolute size, of the β's. This proposition is also illustrated in figure 1, which illustrates donor-country utility as a function of β_i, holding β_j constant – the two kinks in donor utility appear since recipient-country investment is equal to zero ($\alpha = 1$) for high levels of asymmetry between β_i and β_j.

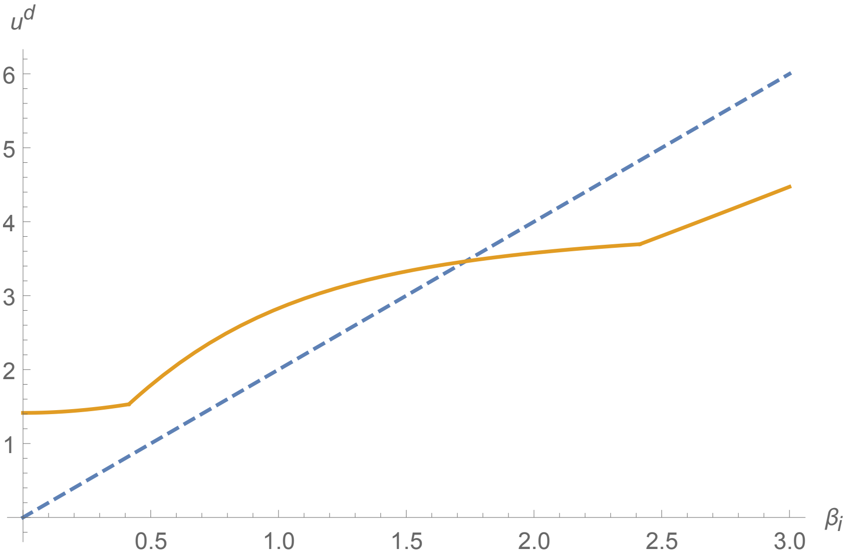


Figure 1: Donor-country i utility under multilateral and bilateral (dashed line) for $x = 4$, $\beta_j = 1$, $\lambda = 0$.

3 Optimal Voting Rule

In the previous section, we assume the fund uses a decision rule of unanimity to determine the allocation of the budget. However, since the decision rule used by the donor countries may impact the investment decisions of the recipient countries, it need not be the case that unanimity is ex ante Pareto optimal (see for example Harstad, 2005, and Maggi and Morreli, 2006). In practice, multilateral funds use a variety of voting rules to make allocation decisions, including unanimity and majority rules (Climate Funds Update, 2015). In this section, we explore the optimal voting rule by characterizing the effect of a majority decision rule on the investment decisions of the recipient countries.

3.1 Model

Here we extend the model to incorporate majority decision making in the spirit of Harstad (2005), where an endogenously chosen majority coalition determines the allocation of the fund. For reasons of tractability and to highlight the effect of majority rule on competition, we consider the case of three donor countries and three partner counties and assume symmetry in spillovers ($\beta_i = \beta_j$).[8]

We model the majority decision rule as follows: After recipient countries choose investment levels and $\{\alpha_j\}$ is revealed, a donor country is randomly chosen as formateur, f (each country has an equal probability of being chosen). The formateur then selects a majority coalition, which bargains over project funding and utility transfers in a manner

[8] This analysis characterizes the optimal q-rule, since in the sub-majority case recipient countries will have no incentive to invest.

analogous to our baseline model. Since the majority coalition is chosen endogenously, the formateur will select the majority coalition that maximizes her expected utility; if the formateur is indifferent regarding which countries to include in the majority coalition, she chooses each country with equal probability. We also assume that utility transfers are restricted to the majority coalition, which implies that countries outside the majority coalition are not fully expropriated. Therefore, the bargaining outcome within the majority coalition is equivalent to the two-country bargaining outcome, given a budget of $3x$.

Additionally, to make the analysis tractable, we consider a stochastic investment technology. Specifically, we restrict each recipient countries' project to be either high quality, $\alpha_j = \alpha^h$, or low quality, $\alpha_j = \alpha^l < \alpha^h$. For ease of exposition, we set $\alpha^l = 1$. The quality of the project is in turn a stochastic function of the level investment chosen by country i, $\delta_j \in [0, 1]$. Specifically:

$$p(\alpha_j = \alpha^h | \delta_j) = \delta_j.$$

Each recipient country faces the same cost function, $c(\delta_j) = \delta_j^2$.

We assume that the indirect benefit of investment continues to be proportional to the level of investment, δ_j. Conceptually, this is consistent with our example of investment in good governance: in the stochastic case, decreasing corruption increases the probability that a recipient country's project realizes as high quality; the decrease in corruption, however, is enjoyed whether or not α_j is equal to α^h or α^l.

Formally, the stochastic model is analogous in expectation to the deterministic model, given that a marginal increase in the recipient country's investment increases the *expected* quality linearly. However, in contrast to the deterministic case, the stochastic model allows for pure strategy equilibria under majority rule, since it avoids an "open set problem" in which recipient countries have an incentive to set marginally higher levels of investments than their peers to ensure that they are chosen to the majority coalition (only mixed-strategy investment strategies exist in equilibrium under majority rule in the deterministic case).

To summarize, the timing of the model is as follows:

1. recipient countries choose investment levels, δ_j.
2. $\{\alpha_j\}$ realizes.
3. A donor country is randomly chosen as formateur, f.
4. The formateur selects a majority coalition, $\mathcal{M}$.
5. The majority coalition bargains over the allocation of project funding and utility transfers.

We restrict the analysis to symmetric equilibria, where all target nations choose the same level of investment, and all donor nations use a symmetric decision rule conditional upon being selected as the formateur.

An equilibrium is defined as follows:

Definition 1
An equilibrium under Majority Rule consists of an investment level, $\delta_j = \delta^m$*, and a decision rule,* ν*, that maps* $\{\alpha_j\}$ *into a majority coalition* $\mathcal{M}$*, where:*

1. *Given* ν *and* $\delta_{k\neq j} = \delta^m$*,* $\delta_j = \delta^m$ *maximizes* $E[g_j - c(\delta_j)|\delta]$ *for each* $j \in P$*.*
2. *Given* $\{\alpha_j\}$*,* ν *maximizes* $u_i(g_j, \{\alpha_j\}, \beta_j\})$ *for each* $i \in D$ *given that* $\{g_i\}$ *is set by NB within the majority coalition.*

That is, for this section, we consider the objective of donor countries to maximize their *expected* utility.

3.2 Analysis

We begin by characterizing the expected allocation to g_j under unanimity in the stochastic investment model:

$$
\begin{aligned}
E[g_j|\delta_j,\{\delta\}]^u = & p(A^h = 0|\delta^u)\Big[\delta_j g^{h,l,l} + (1-\delta_j)g^{l,l,l}\Big] \\
& + p(A^h = 1|\delta^u)\Big[\delta_j g^{h,h,l} + (1-\delta_j)g^{l,l,h}\Big] \\
& + p(A^h = 2|\delta^u)\Big[\delta_j g^{h,h,h} + (1-\delta_j)g^{l,h,h}\Big],
\end{aligned}
$$

where $g^{z,y,w} = (\alpha^z)^2/((\alpha^z)^2 + (\alpha^y)^2 + (\alpha^w)^2)3x$ is the three-country allocation that results from NB.

Next, we consider the expected allocation to g_j under majority. Following backward induction, we first specify the equilibrium decision-rule ν.

Lemma 4
The equilibrium decision-rule, ν*, specifies that the formateur selects a majority coalition,* $\mathcal{M}$*, equal to* $\{f, i\}$*, where* α_i *is the maximum element of the set* $\{\alpha_i\}\backslash\alpha_f$*. If the maximum is not unique,* i *is chosen randomly.*

Since bargaining entails that the majority coalition's surplus is split equally, the formateur maximizes her utility by selecting a majority coalition consisting of herself and the country with highest α_j, since this maximizes the size of the majority coalition's surplus.

Lemma 4 implies that, from country j's perspective, the fund's allocation rule under majority is a function of the number of recipient countries, other than j, that have a high-quality project. We denote this value as $A^h = \sum^{k\in P\backslash j} \mathbb{1}(\alpha_k = \alpha^h)$. This allows us to characterize the expected allocation to g_j as a function of δ_j, given the investment decision of the other two countries (δ^m):

$$\begin{aligned}E[g_j|\delta_j,\{\delta\}]^m =&p(A^h=0|\delta^m)\left(\frac{1}{3}\left[\delta_j g^{h,l}+(1-\delta_j)g^{l,l}\right]+\frac{2}{3}\left[\delta_j g^{h,l}+(1-\delta_j)\frac{1}{2}g^{l,l}\right]\right)\\ &+p(A^h=1|\delta^m)\left(\frac{1}{3}\left[\delta_j g^{h,h}+(1-\delta_j)g^{l,h}\right]+\frac{2}{3}\left[\frac{1}{2}(\delta_j g^{h,h}+\frac{1}{2}(1-\delta_j)g^{l,h})+\frac{1}{2}\delta_j g^{h,h}\right]\right)\\ &+p(A^h=2|\delta^m)\left(\frac{1}{3}\left[\delta_j g^{h,h}+(1-\delta_j)g^{l,h}\right]+\frac{2}{3}\left[\frac{1}{2}\delta_j g^{h,h}\right]\right),\end{aligned}$$

where $g^{z,y}=(\alpha^z)^2/((\alpha^z)^2+(\alpha^y)^2)3x$ is the bargaining solution from the majority coalition, which is equal to the two-country bargaining solution with $\{\alpha^z,\alpha^y\}$.

For each realization of A^h, the above expression divides $E[g_j]$ into the case where $i=j$ is chosen to be the formateur (the first term in brackets on each line) and the case where $i=j$ is not the formateur (the second term in brackets). Note that when $i=j$ is chosen as the formateur, the expression for $E[g_j|\delta_j,\{\delta\}]$ is analogous to the case of unanimity – therefore, the difference between the incentive to invest under the two decision rules stems from the case in which $i=j$ is *not* chosen as the formateur. In this case, the probability that $i=j$ is selected into the majority coalition, and hence receives a positive level of g_j is increasing in δ_j, since Lemma 4 specifies that the formateur will always select the country with a higher level of project quality. This gives j two incentives to invest under majority rule: (1) to increase expected g_j, conditional upon $i=j$ being selected to the majority coalition, and (2) to increase the probability of $i=j$ being selected to the majority coalition.

The addition of incentive (2) under majority rule leads to the following proposition:

Proposition 4
The equilibrium level of investment under majority rule, δ^m, is weakly greater than the level of investment under unanimity rule, δ^u.

Proposition 4 follows from the first-order conditions of $E[g_j|\delta_j,\{\delta\}]$ and is formally proved in the appendix. The result is also illustrated visually in figure 2 for a fixed x ($x=2$).

While Proposition 4 implies that investment levels are higher under majority rule for low levels of α^h, it does not imply that majority rule always outperforms unanimity rule. Instead, under majority rule, there is a tradeoff between higher investment and a utility loss that stems from the fact that majority rule limits funding to the two recipient countries in the majority coalition, and from the concavity of utility over g. This tradeoff is formalized in the following proposition, which considers the utility difference between the two decision rules holding x fixed and varying α^h:

Proposition 5
There exists λ^*, such that iff $\lambda>\lambda^*$, there exist an interval $(\alpha^l,\alpha']$ for some $\alpha'>\alpha^l$ such that the expected utility of the donor countries is higher under majority than unanimity

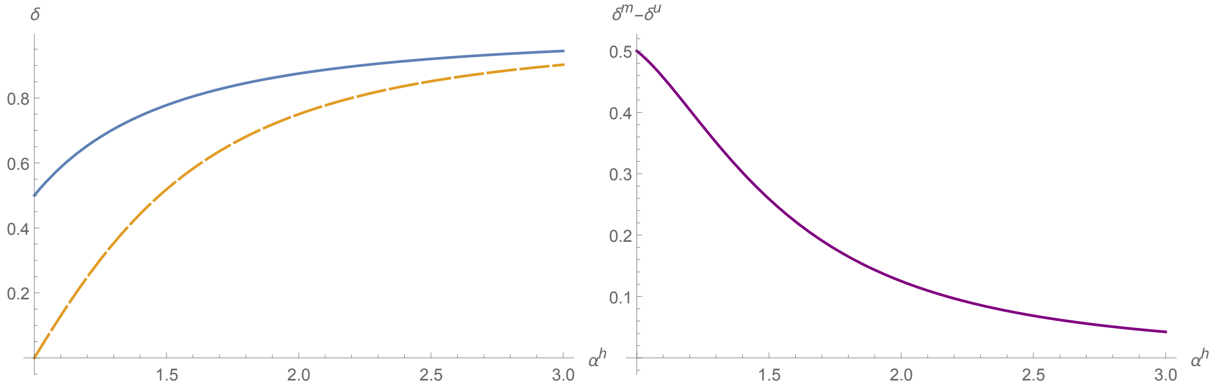
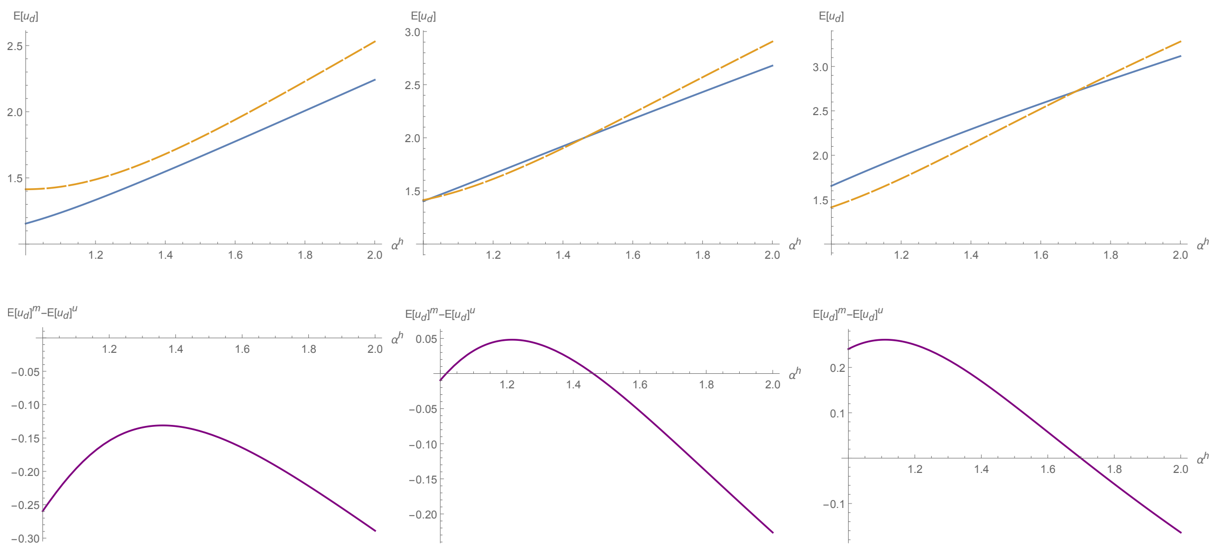


Figure 2: δ^m and δ^u (dashed line) for $x = 2$.

for $\alpha^h \in (\alpha^l, \alpha']$.

The intuition for this result lies in the fact that under unanimity rule, as $\alpha^h \to \alpha^l$, the incentive to invest approaches zero for the recipient countries, since g^j approaches x for any α^j. Under majority rule, however, the incentive to invest stays strictly positive, since any country with $\alpha^j = \alpha^l$ is more likely to be left out of the majority coalition and receive $g^j = 0$. Therefore, as $\alpha^h \to \alpha^l$, $\delta^u \to 0$ while $\delta^m \to 1/4x > 0$. However, as $\alpha^h \to \alpha^l$, the direct benefit of investment (to the donor countries) also approaches zero, while the utility cost of restricting the budget allocation to two countries stays strictly positive. Therefore, for majority rule to dominate unanimity at low α^h, it must be the case that the indirect benefit of investment outweighs this utility cost.



$\lambda = 0$. $\lambda = 0.5$. $\lambda = 1$.

Figure 3: $E[u_d]^m$ and $E[u_d]^u$ (dashed line) for $x = 2$.

This result is also illustrated by figure 3, which shows the utility difference under the two voting rules for different values of λ. Note that utility difference between majority rule and unanimity rule is the highest at an interior value of α^h. The utility difference is increasing initially since, as α^h increases, the direct benefit of higher investment increases. However, as α^h increases, the difference in the investment levels under majority and unanimity also decreases.[9] Therefore, for high enough levels of α^h, the utility difference between the two decision rules is decreasing, resulting in an interior maximum.[10]

Lastly, we interpret these results in more general context: In the stochastic model, decreasing α_h corresponds to decreasing the benefit of higher investment. Therefore, the result that majority rule can dominated unanimity rule for low levels of α^h corresponds to the statement that majority rule is the optimal voting rule when the direct benefit of partner-country investment is low. The intuition behind this result is straightforward: when the direct benefit of higher investment to the donor countries is relatively low, then the incentive to invest that is generated by collective allocation is relatively weak. Therefore, a majority rule is preferable in these cases, since it provides an additional incentive for recipient countries to invest to ensure that their donor country is selected to the majority coalition.

4 Conclusion

In this paper, we consider a formal model of the allocation of aid spending in an environment where donor countries face a bias over which recipient countries receive funding. Our analysis provides several important insights regarding the optimal design of multilateral aid organizations. As highlighted by Svensson (2000, 2003), multilateral aid organizations must focus on distributing aid in a manner that provides an incentive for developing nations to invest in reform. However, given competing national and special party interests, the question is how to enforce this objective. Here, we show that competition in the area of reform can arise endogenously when donor countries directly bargain over the allocation of aid funds, and that this competition is intensified under majority rule, as recipient countries invest in reform to increases the probability that their project will be selected by the endogenous majority coalition.

We emphasize that the predictions of our model only apply to multilateral aid funds that allocate aid spending via an unstructured bargaining process. In recent years, "earmarked" donations (aka multi-bi aid) have become increasingly common as donor countries seek to take advantage of the benefits of scale of international organizations, while

[9] At high enough levels of α^h; by L'Hôpital's Rule, both δ^m and δ^u approach $\min\{1, 2x/(2+x)\}$ as $\alpha^h \to \infty$.

[10] Also, note that figure 3 demonstrates that a range of α^h where majority outperforms unanimity need not exist. In fact, simulations show that if $\lambda = 0$, then unanimity is preferable to majority for all x, α^h. With a higher budget, the benefit of higher partner-county investment increases; however, the incentive to invest also increases, decreasing the difference in investment levels between majority and unanimity – with symmetric β_js and $\lambda = 0$, the later effect dominates and unanimity is optimal for the donor countries.

ensuring that aid is distributed according to national priorities. However, as our paper shows, earmarking diminishes the incentive of recipient countries to invest in reforms, since it circumvents multilateral bargaining. Therefore, in this case, less structure can result in greater efficiency.

Lastly, in future research we hope to consider the role of competition between multilateral organizations. Given the proliferation of climate change aid funds, competition has arguably arisen over funding the best projects. While the effect of such competition may be beneficial when the quality of the set of projects is exogenous, the effect of competition on endogenous quality is unclear. More research is needed to clarify the effect of the "market structure" of multilateral organizations on recipient countries' incentives to invest in reform.

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Appendix

A Proofs

A.1 Proofs for Section 2

Proof of Lemma 1: Since country i does not value spending on g_j, it has no incentive to spend there. There is also no alternative productive use for the aid budget, so all x_i is spent on g_i. ■

Proof of Lemma 2: The first order conditions for problem 4 are:

$$\frac{\alpha_i \beta_i}{\sqrt{g_i}} = \nu \quad \forall i \tag{13}$$

$$g_1 + g_2 = x \tag{14}$$

where ν is the Lagrange multiplier on the budget constraint. They yield the optimal allocation rule in the fund:

$$g_i = \frac{(\alpha_i \beta_i)^2}{(\alpha_1 \beta_1)^2 + (\alpha_2 \beta_2)^2} 2x \tag{15}$$

$$g_1 + g_2 = 2x. \tag{16}$$

■

Proof of Lemma 3: Since reforms are costly and do not change the allocation of aid, target countries choose the minimum level $\alpha_j = 1$. ■

Proof of Proposition 1: Target countries move simultaneously, so we are looking for Nash equilibria. Each target country solves the stated problem, taking as given the other's reform investment level. This gives us two best response functions:

$$1 = 2x\alpha_i^2 \frac{(\beta_1 \beta_2)^2}{((\alpha_1 \beta_1)^2 + (\alpha_2 \beta_2)^2)^2} \quad \forall i, \tag{17}$$

Setting these two best response functions, for $j = i, k$, equal to each other and simplifying gives the result that $\alpha_1 = \alpha_2 = \alpha$, and thus

$$\alpha = \sqrt{2x}\frac{\beta_1\beta_2}{\beta_1^2 + \beta_2^2} \tag{18}$$

There exist parameter combinations for which $\alpha < 1$, in which case target countries must choose the minimum investment level $\alpha_i = 1$. This particularly concerns small budgets of $x < 2$. There clearly also exist parameter combinations where $\alpha > 1$, so that it can be concluded that investment is always weakly larger than under bilateral allocation. ■

Proof of Corollary 1: The partial derivative of α with respect to x is positive. Since the true implemented α cannot fall below the minimum required investment of 1, α is not increasing in x until $x > 2$. ■

Proof of Corollary 2: Rewrite α in terms of the ratio $B = \frac{\beta_1}{\beta_2}$:

$$\alpha = \sqrt{2x}\frac{B}{1 + B^2} \tag{19}$$

α is maximized when $B = 1$. The necessary and sufficient conditions, respectively, are:

$$\frac{\partial\alpha}{\partial B} = \sqrt{2x}\frac{1 - B^2}{(1 + B^2)^2} = 0 \tag{20}$$

$$\rightarrow B = 1 \tag{21}$$

and

$$\frac{\partial^2}{\partial^2 B} = \sqrt{2x}\frac{-2B(1 + B^2)^2 - (1 - B^2)2(1 + B^2)2B}{(1 + B^2)^4} \tag{22}$$

$$\rightarrow \text{at } B = 1 \quad \frac{\partial^2}{\partial^2 B} < 0 \tag{23}$$

For $B < 1$, an increase in B corresponds to a decrease in asymmetry, for $B > 1$ an increase in B corresponds to an increase in asymmetry. ■

Proof of Proposition 2: Replacing $\beta_1 = \beta_2 = \beta$ in equation (12) yields the surplus for each donor nation from joining the fund:

$$F = \frac{1}{2}\alpha\sqrt{2x}\frac{2\beta^2}{\sqrt{2\beta^2}} + \lambda(\alpha - 1) - \beta\sqrt{x} \tag{24}$$

$$= (\sqrt{x}\beta + \lambda)(\alpha - 1) \geq 0, \tag{25}$$

since $\alpha \geq 1$. ■

Proof of Proposition 3: First, we consider the case where $\alpha = 1$ ($\delta = 0$). In this case, there is no advantage from the fund in terms of increased investment incentives. Without loss of generality, assume $\beta_i \geq \beta_j$. At $\beta_i = \beta_j$, the allocation of funds will be exactly the same as under bilateral aid:

$$g_i = 2x \frac{\beta_i^2}{\beta_i^2 + \beta_j^2} \quad (26)$$

$$\beta_1 = \beta_2 \to g_i = x, \quad (27)$$

so that both countries should be indifferent between the two forms of aid allocation. Indeed, for $\alpha = 1$ and $\beta_1 = \beta_2$

$$F_i = \frac{1}{2}\alpha[\beta_1\sqrt{g_1} + \beta_2\sqrt{g_2}] - \beta_i\sqrt{x} \quad (28)$$

$$= \frac{1}{2}2\beta\sqrt{x} - \beta\sqrt{x} = 0. \quad (29)$$

It then remains to be shown that the surplus from joining a fund, F, is larger than zero for country i whenever $\beta_i < \beta_j$ and smaller than zero otherwise. To see that, it suffices to show that the derivative of F with respect to β_i is less than zero everywhere:

$$\frac{\partial F_i}{\partial \beta_i} = \frac{1}{2}\sqrt{2x}\frac{2\beta_i\sqrt{\beta_i^2 + \beta_j^2} - \frac{\beta_i(\beta_i^2+\beta_j^2)}{\sqrt{\beta_i^2+\beta_j^2}}}{\sqrt{\beta_i^2 + \beta_j^2}^2} - \sqrt{x} \quad (30)$$

$$= \sqrt{x}\left[\frac{\sqrt{2}}{2}\frac{\beta_i}{\sqrt{\beta_i^2 + \beta_j^2}} - 1\right] < 0 \quad (31)$$

$$(32)$$

Thus, when $\alpha = 1$ ($\delta = 0$), the donor country with the higher bias (i) will prefer to allocate aid bilaterally, and set $f_i = 0$.

Next, note that since $\alpha = \max\{1, \sqrt{2x}\frac{\beta_i\beta_j}{\beta_i^2+\beta_j^2}\}$, and since:

$$\lim_{\beta_i \to \infty} \sqrt{2x}\frac{\beta_i\beta_j}{\beta_i^2 + \beta_j^2} = 0,$$

by L'Hôpital's Rule, there exists β' such that $\beta_i \geq \beta'$ implies $\alpha = 1$, which proves the result. ■

A.2 Proofs for Section 3

Proof Lemma 4: First note that the formateur will always select herself to the majority coalition, since the payoff of belonging to the majority coalition is strictly positive and utility transfers are restricted to the majority coalition. Since NB prescribes an equal split of the utility surplus, the utility of the formateur is equal to $1/2S$, where $S = \sum_{i\in M} \alpha_i g_i^{NB}$. And since NB results in the set of $\{g_j\}$ that maximizes S, $S(\{\alpha_f, \alpha^h\}) > S(\{\alpha_f, \alpha^l\})$. This implies that f has a strictly dominant strategy of selecting $M = \{f, i\}$ with $\alpha_i = \alpha_h$ over $M = \{f, i'\}$ with $\alpha_{i'} = \alpha_l$. Lastly, by assumption, f randomizes $i \in M, i \neq f$ if the maximal element of $\{\alpha_i\}\backslash\alpha_f$ is not unique. ■

Proof Proposition 4: Each partner country sets δ_j to maximize its expected utility, equal to:

$$E[u(\delta_j, \{\delta\})]^q = E[g_j|\delta_j, \{\delta\}]^q - \delta_j^2,$$

where q corresponds to the decision rule. The expected allocations are repeated here for convenience.

$$\begin{aligned} E[g_j|\delta_j, \{\delta\}]^u =& p(A^h = 0|\delta^u)\Big[\delta_j g^{h,l,l} + (1-\delta_j)g^{l,l,l}\Big] \\ &+ p(A^h = 1|\delta^u)\Big[\delta_j g^{h,h,l} + (1-\delta_j)g^{l,l,h}\Big] \\ &+ p(A^h = 2|\delta^u)\Big[\delta_j g^{h,h,h} + (1-\delta_j)g^{l,h,h}\Big], \end{aligned}$$

$$\begin{aligned} E[g_j|\delta_j, \{\delta\}]^m =& p(A^h = 0|\delta^m)\left(\frac{1}{3}\Big[\delta_j g^{h,l} + (1-\delta_j)g^{l,l}\Big] + \frac{2}{3}\Big[\delta_j g^{h,l} + (1-\delta_j)\frac{1}{2}g^{l,l}\Big]\right) \\ &+ p(A^h = 1|\delta^m)\left(\frac{1}{3}\Big[\delta_j g^{h,h} + (1-\delta_j)g^{l,h}\Big] + \frac{2}{3}\Big[\frac{1}{2}(\delta_j g^{h,h} + \frac{1}{2}(1-\delta_j)g^{l,h}) + \frac{1}{2}\delta_j g^{h,h}\Big]\right) \\ &+ p(A^h = 2|\delta^m)\left(\frac{1}{3}\Big[\delta_j g^{h,h} + (1-\delta_j)g^{l,h}\Big] + \frac{2}{3}\Big[\frac{1}{2}\delta_j g^{h,h}\Big]\right), \end{aligned}$$

After taking the first-order conditions of $E[u(\delta_j, \{\delta\})]^q$, imposing symmetry in the investment decisions, and simplifying, we get the following equation, which implicitly characterizes an interior solution for δ^u:

$$2\delta = (1-\delta^u)^2\big(g^{hll} - x\big) + 2(1-\delta^u)\delta^u\big(g^{hhl} - g^{llh}\big) + (\delta^u)^2\big(x - g^{lhh}\big). \tag{33}$$

And for δ^m:

$$2\delta = (1-\delta^m)^2\left(g^{hl} - x\right) + 2(1-\delta^m)\delta^m\left(\frac{1}{6}g^{hl} + x - \frac{1}{2}g^{lh}\right) + (\delta^m)^2\left(x - \frac{1}{3}g^{lh}\right). \tag{34}$$

Equation 33 implies that δ^u is characterized by:

$$\delta^u = \min\left\{1, \frac{((\alpha^h)^2-1)(2(\alpha^h)^2+1)x}{2+5(\alpha^h)^2+2(\alpha^h)^4+((\alpha^h)^2-1)^2x}\right\}, \tag{35}$$

while equation 34 implies that δ^m is characterized by:

$$\delta^m = \min\left\{1, \frac{(2(\alpha^h)^2-1)x}{2+2(\alpha^h)^2+((\alpha^h)^2-1)x}\right\}. \tag{36}$$

Deriving an analytical proof that $\delta^m > \delta^u$ is cumbersome, however, equations 35 and 36 can easily be used to verify the result numerically (code available on request). ■

Proof Proposition 5: Define $\delta^d(\alpha^h)$ as:

$$\delta^d(\alpha^h) = \delta^m(\alpha^h) - \delta^u(\alpha^h).$$

Since both equation 35 and equation 36 are continuous at $\alpha^h = 1$, it follows that $\lim_{\alpha^h \to \alpha^l} \delta^d(\alpha^h) = \delta^d(1)$, which simplifies to $\delta^d(1) = x/4$. Moreover, as $\alpha^h \to \alpha^l$, the NB allocations, $g^{z,y,w}$ and $g^{z,y}$, approach x and $3/2x$, respectively, regardless of $\{\alpha_j\}$.

Together, these two results imply that the difference in the donor country's expected utility under majority and unanimity, as $\alpha^h \to \alpha^l$ is equal to:

$$E[u^d]^m - E[u^d]^u = -\alpha^l\left[x^{\frac{1}{2}} - \frac{2}{3}\left(\frac{3}{2}x\right)^{\frac{1}{2}}\right] + \lambda\left(\frac{x}{4}\right),$$

where the first term reflects the utility loss from allocating X over two projects only, and the second term reflects the indirect utility benefit of the higher investment under majority rule (the direct benefit approaches zero as $\alpha^h \to \alpha^l$). Clearly, donors' expected utility is greater under majority rule in a neighborhood of $\alpha^h = \alpha^l$ if, and only if:

$$\lambda > \frac{4\alpha^l\left[1 - \frac{2}{3}\left(\frac{3}{2}\right)^{\frac{1}{2}}\right]}{x^{\frac{1}{2}}}.$$

■