I am leaving EU: the political economy of exit clauses

Martijn Huysmans[1]

Advisor: Christophe Crombez[2]

December 16, 2016

Abstract

This article presents a political economic analysis of exit clauses in currency unions and (con)federations. In such unions, members' benefits depend on their characteristics and on the state of the world. If a member's benefits become negative, it may wish to exit. Using a dynamic stochastic model based on real option theory, we show that state-contingent exit penalties can enable socially efficient dissolution of the union. Even if exit penalties cannot be made state-contingent, they may still enhance social welfare by preventing secession wars. This finding runs counter to the dominant point of view in the literature that exit clauses should be avoided in federations. To demonstrate the validity of the theory, we show how it can explain the dynamics of the breakup of Yugoslavia.

Keywords: Exit Clauses, Withdrawal, Secession, Real Options

[1] Doctoral Candidate, LICOS Research Center, Faculty of Economics and Business, KU Leuven. Email: martijn.huysmans@kuleuven.be.

[2] Professor of Political Economy, Faculty of Economics and Business, KU Leuven, and Consulting Professor, Freeman Spogli Institute for International Studies, Stanford University.

Introduction

In this article we develop a political economic model of exit clauses in political unions, by which we mean currency unions such as the Euro and (con)federations such as the United States or the European Union (EU). An exit clause specifies the conditions of unilateral exit or withdrawal from an agreement. Examples of exit clause conditions are the payment of a penalty, waiting out a notice period, or obtaining consent from (a certain fraction of) the remaining parties. Exit clauses are a common feature of contracts among individuals and firms. Rental agreements and employment contracts typically specify notice periods and sometimes fines for early contract termination. Another example are pre-nuptial or pre-marital contracts, which may specify monetary compensation in the event of a divorce (Rainer, 2007).

In most types of international treaties, so-called withdrawal clauses are relatively prevalent (Koremenos, 2016). This is not the case for exit clauses in political unions: exit clauses are not common in currency unions and (con)federations. To the best of our knowledge, the only current examples are the secession clauses in the constitutions of Ethiopia (Habtu, 2005) and Saint Kitts and Nevis (Weinstock, 2001), and the withdrawal clause in the Treaty on European Union (Athanassiou, 2009).[3] Liechtenstein, a non-federal country, also has a constitutional secession clause for its municipalities (Sorens, 2016).

An historical example of an exit clause in a political union is the 1885 liquidation clause in the Latin Monetary Union (B. J. Cohen, 1993). The Latin Monetary Union was a currency union created in 1865 between Belgium, France, Italy and Switzerland. This liquidation clause is particularly interesting because it specified an exit penalty: the clause required states wishing to exit from the monetary union to buy back all their silver held at the remaining members' banks (silver had lost a lot of value relative to gold). Perhaps thanks to this costly exit clause, the Latin Monetary Union persisted until Switzerland's withdrawal in 1926, and was dissolved in 1927 (B. J. Cohen, 1993). Another example is Article 72 in the constitution of the

[3] The conditions in the Federal Republic Ethiopia include a two-thirds majority in the Parliament of the seceding region, and a simple majority in a regional referendum to be organized by the federal state within three years. The Federation of Saint Kitts and Nevis allows for secession in case a two-thirds majority is reached in an independence referendum in the seceding region. Article 50 of the Treaty on European Union specifies that any Member State may withdraw from the Union by notifying the Council, and is guaranteed the right to leave unilaterally within two years.

Soviet Union (Sunstein, 1991: 645-647): "Each Union Republic shall retain the right freely to secede from the USSR". More recently, two historical examples are the 2005 Comprehensive Peace Agreement in Sudan[4], and Article 60 of the 2003 Constitutional Charter of Serbia and Montenegro. We discuss the case of Montenegro in detail in the empirical section on Yugoslavia.

For the purpose of studying exit clauses, political unions are different from common contracts in two important ways. First, for common contracts such as the lease of an apartment or an employment contract, legislators have typically defined default exit rules that are applicable even if there are none in the contract, or have limited the scope for freely specifying exit conditions. Such default exit rules are not applicable to political unions, although the Vienna Convention on the Law of Treaties (VCLT) does provide some ground rules about withdrawal from international agreements.[5] Second, common contracts are enforceable before national courts and political unions are not. Depending on the reputational costs, unilateral exit from political unions may occur, for instance through a secession war (Helfer, 2005). Conversely, attempts at exit on the basis of an exit clause may be met with anti-secession wars, as happened in the break-up of the Soviet Union.

Table 1 gives an empirical illustration of the relationship between exit, exit-related violence and exit clauses. The left column collects currency unions or (con)federations without exit clauses, while the right column lists examples with exit clauses. The first two rows gives some notable examples of violence associated with exit, while the third row gives examples of peaceful exits, and the fourth row of unions from which exit has so far not occurred.

[4] The 2005 Comprehensive Peace Agreement ended the Second Sudanese Civil War which ran from 1983 to 2005 between the Sudanese government and the Sudan People's Liberation Army located in the South of the country. The peace treaty stipulated that the South would have autonomy for 6 years, and that oil revenues would be split equally during this period. The period of autonomy would be followed by an independence referendum in 2011. In the referendum, 98.8% voted in favor of independence, and South Sudan declared its independence.

[5] Article 56 of the VCLT specifies that unilateral withdrawal from a treaty is only allowed if the treaty explicitly specifies a withdrawal right, or if a withdrawal right was implied by the parties or the nature of the treaty. If withdrawal is allowed, the default notice period is twelve months. See Koremenos (2016: 144) for an empirical investigation of states' actual practice regarding notice periods, which diverges from the default twelve months in more than half of her sample. If there is no explicit nor implicit withdrawal right, article 62 of the VCLT provides for the possibility of withdrawal under "fundamental change of circumstances", although legal scholars are divided about the validity and applicability of this so-called *rebus sic stantibus* clause in practice (Helfer 2005: 1643-1644).

	No exit clause	**Exit clause**
Violence without exit	ETA Basque terrorism (1961-2011) Second Sudanese Civil War (1983-2005) US Civil War (1861-1865)	-
Violent exit	Yugoslavia-Bosnia Herzegovina (1992) Pakistan-Bangladesh (1971) United Arab Republic-Syria (1961) Netherlands-Belgium (1830)	Dissolution of Soviet Union (1990-1991)
Peaceful exit	Czech Republic & Slovakia (1993) Scandinavian Monetary Union (1931) Sweden-Norway (1905)	Sudan-South Sudan (2011) Serbia-Montenegro (2006) Latin Monetary Union-Switzerland (1926)
No exit so far	Spain-Catalonia United Kingdom-Scotland Eurozone-Greece Canada-Québec	European Union (EU) Ethiopia Saint Kitts & Nevis

Table 1. Historical cases of exit, exit-related violence and exit clauses

Examples of exit-related violence occurred during the terrorist period of the Basque independence movement ETA in Spain, the Second Sudanese Civil War, the US Civil War, the independence of Bosnia-Herzegovina from Yugoslavia, the independence of Bangladesh (formerly East-Pakistan) from Pakistan, the exit of Syria from the United Arab Republic which it formed with Egypt, the independence of Belgium from the Netherlands, and the dissolution of the Soviet Union.

Formal exit clauses can enable peaceful exit, as in the independence of South Sudan from Sudan, the independence of Montenegro from Serbia-Montenegro, and the exit of Switzerland from the Latin Monetary Union. However, a pre-agreed exit clause is not a requirement for peaceful exit: the "velvet divorce" of Czechoslovakia, the dissolution of the Scandinavian Monetary Union (B. J. Cohen, 1993), and the independence of Norway from Sweden occurred peacefully. While having no exit clause is not a guarantee

against exit, having an exit clause of course does not imply that exit is bound to occur, as the examples of Ethiopia and Saint Kitts & Nevis show. Britain has voted to leave the EU on 23 June 2016, but has so far not triggered Article 50, which would be required to actually start the process of exit. No doubt the largest category in the universe of political unions are those which have no exit clause and from which no exit has occurred. Well-known examples are Spain, the United Kingdom, the Eurozone, and Canada.

If anything emerges from Table 1, it is that there is no immediately obvious empirical relationship between exit clauses, exit, and violence. This is where we believe lies the value of the model we develop in this article: it identifies theoretically the expected relationships between these elements. We identify three modes of exit: exit based on an exit clause, unilateral exit, and negotiated exit. Our main finding will be that appropriately costly exit clauses may ex-ante enable efficient exit, if the costs of an anti-secession war are sufficiently high. These costs comprise both tangible costs like military expenses, and reputation costs, which depend on regime type. During the breakup of the Soviet Union, the reputation costs of conducting anti-secession wars were arguably small in comparison to the reputation costs the EU and its Member States would suffer if it tried preventing Brexit forcefully. Without exit clauses, negotiating exit ex-post may be difficult, hence the numerous historical examples of (violent) unilateral exit.

This paper is structured as follows. First we introduce the notion of efficient breach and review the literature on exit clauses. Then we introduce our own model, of which we formally solve a decision theoretical version. Based on the insights from this analysis, we discuss the game-theoretical aspects of exit. Because a formal game-theoretical model incorporating barriers to negotiated exit would not be tractable, we offer a discussion instead. We illustrate the validity of our conclusions by showing how the dynamics of the breakup of Yugoslavia can be explained based on our theory. In the conclusion, we identify four tentative reasons why exit clauses are actually rare in currency unions and (con)federations.

Efficient breach and the literature on exit clauses

Without exit clauses, economically inefficient contracts may be upheld. In some states of the world the aggregate benefit from the contract may be negative while one or more participants still benefit. Such

"vested interests" may be opposed to renegotiation (Aghion & Bolton, 2003). This may be true even if ex ante, before the state of the world is known, all parties would have prefered allowing socially efficient reforms (exit being an extreme type of reform).

The idea that not respecting a contract may be socially desirable is known as "efficient breach" (Goetz & Scott, 1977). In theory, if exit is efficient those better off outside the contract could offer an acceptable side-payment for exit and make everyone better off – an application of Coase (1960). In this light, Drèze, De Grauwe, & Edwards (1993) discuss a practical rule suggested by Drèze to ex-post reapportion national debt at the time of secession. However, as pointed out by De Grauwe, ex-post renegotiation may be very difficult politically. Hence there may be a role for ex-ante negotiated exit clauses to enable efficient breach.

Exit from a (con)federation specifically is usually called secession. Depending on how a secession takes place, it can be peaceful and swift, or painful and costly (Young, 1994). Having constitutional exit provisions can expedite the process once it has become inevitable, increasing order and predictability. Carefully constructed exit clauses may even accomplish this goal without necessarily increasing the likelihood of secession (Weinstock, 2001). By contrast, without exit provisions, exit boils down to a fundamental contract renegotiation, implying that unanimity or the use of threats, coercion or violence are needed to change the status quo. If unanimity cannot be reached, embarking on a destructive secession war (figuratively or literally) may be the only remaining alternative for the member wishing to secede.

In spite of the potential benefits of secession clauses, the dominant position in the constitutionalist literature is that they should be avoided. This position was pioneered by Sunstein (1991, 2001), who sees constitutions as pre-commitment strategies in the presence of multiple equilibria. He argues that if there are multiple equilibria in the federation game, equilibria with secession must be Pareto inefficient. Having a constitution that precludes secession then functions as a pre-commitment device to a Pareto superior equilibrium. Chen & Ordeshook (1994) take the same starting point as Sunstein, but develop a formal three-player game-theoretical model which is taken up further in Filippov, Ordeshook, & Shvetsova (2004). Their conclusion is that there should be a constitutional ban on exit, in order to coordinate on non-secession equilibria, which are assumed to be Pareto superior.

The strength of the conclusions drawn by Chen & Ordeshook (1994) is limited because of three strong assumptions. First, they assume that maintaining the federation is always socially efficient (efficient breach does not exist by assumption). Second, they assume that without an exit clause, exit cannot occur. Of course in reality a party wishing to exit may embark on a destructive secession war (figuratively or literally). Such unregulated exit may be particularly chaotic and costly. Third, they assume that exit clauses are necessarily unconditional, i.e. free in use. In fact, exit will necessarily cause one-off transaction costs, and exit clauses may stipulate costly conditions such as the payment of an exit penalty or the organization of a referendum.

Bordignon & Brusco (2001) analyze optimal secession rules in a two-player game with two periods and a discrete stochastic state. Their model, like ours, covers both (con)federations and currency unions. In period one, members may receive some benefit that depends on the expected stability in period two. This feature of the model captures the fact that the benefits of currency unions (e.g. price stability, increased foreign investment) depend on their perceived stability. In period two, members of the federation decide between accepting a federal allocation or starting a secession war. Bordignon & Brusco find that secession clauses may be inefficient if the benefits from the federation depend significantly on its perceived stability. While their model allows for stochastic shocks and hence the possibility of efficient breach, it still has some important limitations. The discretized set-up with only two periods and four states of the world does not allow for dynamics such as enduring short-term losses. In addition, they introduce a new restrictive assumption compared to Chen & Ordeshook (1994), namely that countries are ex-ante identical.

Using a formal model of costly exit clauses, we show that the position of Sunstein (1991) and Chen & Ordeshook (1994) against exit clauses is untenable. By modeling the dynamic aspect of exit decisions in continuous time, we introduce considerations of optimal exit timing absent in Bordignon & Brusco (2001). In addition, our model allows for members of the union to be different ex-ante, so that the distributional consequences of exit clauses can be studied. We also show that appropriately costly exit clauses do not increase the likelihood of exit given the implicit option of unilateral exit through a secession war.

The model

The following continuous time model describes a political union between two members indexed by $i \in \{A, B\}$. For each member i, the net benefits of such a union will depend on its nature (i.e. a currency union or a federation), and on the member's characteristics. For instance, in the case of a federal country, the net benefits will be roughly composed of economies of scale in the provision of public goods, the internalization of externalities, fiscal transfers, and welfare losses from centralized decision-making in the presence of heterogeneity between the federated members (Alesina & Spolaore, 2003; Desmet, Breton, Ortuño-Ortin, & Weber, 2011). Each of these components will be determined by the substantive terms of the agreement, such as the rules for computing fiscal transfers, and the characteristics of the members, such as their average income. We denote the net benefits at the time of the agreement by a member's type, θ_i. A high type indicates high benefits from the union.

However, over time the benefits may change because of changes in the state of the world. For instance, a member may become richer. While this may be good for the member per se, it may also decrease its benefits from the federation, because it will have to pay higher fiscal transfers. This seems to be case, for instance, in Catalonia, Scotland and Flanders. In our model, we capture such changes in benefits by a member's state x_i. A high state means that a member is currently enjoying the union more, and vice versa. We assume that the link between the state of the world and benefit flows is fully exogenous. Substantively, this assumption means that the terms of the agreement cannot be renegotiated: the rules for computing fiscal transfers cannot be changed. While this is a limitation, we leave for future work a model where the link between the state of the world and benefits is endogenous.

To summarize, the net benefit flow from the agreement at time t for both members depends on the stochastic state of the world $x(t) = \big(x_A(t), x_B(t)\big)$ and on the members' types θ_i. Concretely, member i's net benefit flow $\pi_i(t)$ at time t is given by:

$$\pi_i(t) = x_i(t) + \theta_i \qquad (1)$$

The state of the world captures the changes in benefits over time, which are uncertain. For simplicity, assume that member i's state $x_i(t)$ evolves according to a Brownian motion without drift and with variance

σ_i^2. A Brownian motion is a mathematical representation of a random process over time. Over any given time period dt, it can go up or down. The increments dx_i have a normal distribution characterized by variance σ_i^2 (Mörters & Peres, 2010). A higher variance σ_i^2 means that member i's benefits are expected to fluctuate more. Formally, a Brownian motion is characterized by

$$dx_i = \sigma_i \epsilon_{it} \sqrt{dt} \tag{2}$$

where ϵ_{it} has a standard normal distribution and is serially uncorrelated so that for all $t > 0$

$$E[\epsilon_{it}] = 0, \qquad Var(\epsilon_{it}) = E\left[\epsilon_{it}^2\right] = 1, \qquad E\left[\epsilon_{it_1}\epsilon_{it_2}\right] = 0, t_1 \neq t_2 \tag{3}$$

This implies that $E[dx_i] = 0$ and $E[(dx_i)^2] = \sigma_i^2 dt$: benefits are expected to remain the same, but have a variance of σ_i^2 per unit of time.

Assume furthermore that the increments ϵ_{At} and ϵ_{Bt} of the Brownian motions $x_A(t)$ and $x_B(t)$ are jointly normally distributed for all t, with correlation ρ so that

$$E[\epsilon_{At}\epsilon_{Bt}] = \rho, \; E[dx_A dx_B] = \rho \sigma_A \sigma_B dt \tag{4}$$

A positive correlation means that both members' benefits tend to move together. A negative correlation means that changes in the state of the world tend to have an opposite effect on both members. This would be the case, for instance, for fiscal transfers. If one member grows richer, it will benefit less from the union while the other will benefit more, since transfers will increase.

Without loss of generality, suppose that the agreement starts at $t = 0$ and set $x_i(0) = 0$. Since we have assumed no drift in the states x_i, their ex ante expected value is 0 for any future date T; mathematically, $E[x_i(T)] = 0, \forall T > 0$. Similarly, the expected value at time t for a later time T is simply the value at the time of the expectation: $E_t[x_i(T)] = x_i(t)$. The variance of a Brownian motion increases linearly over time. At time t the probability density for time T at state x_i is

$$P_t(x_i(T) = x_i) = \phi\big(x_i; x_i(t), \sigma_i^2(T-t)\big) = \phi\left(\frac{x_i - x_i(t)}{\sigma_i\sqrt{T-t}}\right) \tag{5}$$

where $\phi(x;\mu,\sigma^2)$ is the probability density function (pdf) of the normal distribution with mean μ and variance σ^2, and $\phi(x)$ is the pdf of the standard normal distribution. The expected change in x_i is zero, large changes are less likely than small changes, but the expected magnitude of changes increases over time.

Assume that the members have a common discount rate r, so that the expected discounted benefit from the agreement for member i until time T is $E\left[\int_0^T \pi_i(t)e^{-rt}dt\right]$. The ex-ante expected value for member i from a perpetual agreement is

$$E\left[\int_0^{+\infty} \pi_i(t)e^{-rt}dt\right] = \lim_{T\to+\infty}\left(\frac{\theta_i}{r} - \frac{\theta_i}{r}e^{-rT}\right) = \frac{\theta_i}{r} \qquad (6)$$

Hence, member i will only voluntarily enter the agreement if its type $\theta_i > 0$. However, even if the agreement is ex ante positive, in some states of the world $x(t)$ it may turn out bad for one or both members, so that they may wish to exit from the agreement.

In the model, we will consider three types of ex-ante specified exit conditions: penalties, one-off costs and notice periods. An exit penalty consists of the payment of a penalty c_i by the party exiting i to the remaining party j. All other costs associated with exit other than the penalty are grouped in the one-off costs k_i. This parameter comprises all non-transfer costs required by the exit clause, whether stated explicitly or required implicitly. For instance, in the case of a federation the one-off costs comprise the legal costs of writing a new constitution, negotiating international treaties, and the logistical costs of organizing a referendum (if required). For a monetary union, the one-off costs comprise the cost of market uncertainty and the costs of printing new money. The costs c_i and k_i have the same effect on party i, but only c_i will be received by j as a compensation for exit. While k_i is partly endogenous, c_i can be freely specified. A notice period specifies the minimal amount of time N between a member's announcement that it wishes to exit, and the effective exit. We analyze exit penalties and one-off exit costs in the main text. In the Appendix, we show that notice periods do not affect decisions of exit timing, although they do affect the members' expected benefits from the agreement.

As an example, consider the EU's Article 50. This clause does not specify a penalty. An example of a penalty that could conceivably have been included would be the payment of an additional year's budget contribution. The clause does not specify one-off costs such as the organization of a referendum (this is left to Member States' internal decision-making rules), but clearly exit from the EU requires a substantial amount of legal costs. While the article does not specify a penalty, it does specify a waiting period of two years. Only two years after triggering Article 50 is an EU Member State guaranteed to be able to exit the EU. The clause gives no guarantees on the nature of future relations with the EU. By calling Article 50, a Member State can only guarantee full departure from the EU. This means losing all of the associated benefits, such a access to the Single Market, and all of the costs, such as the budget contribution. A leaving Member State may attempt to negotiate more favorable terms with the rest of the EU, but such negotiations are theoretically distinct from the exit. By insisting that the EU is a package deal – and in particular that Single Market access cannot be separated from labor mobility – other EU Member States emphasize this point.

Optimal exit in a decision theoretical setting

To analyze the consequences of exit clause, we start by formally solving a decision-theoretical version of the model, in which only one member can exit. In the next section we discuss game-theoretical considerations. We do not formally solve a game-theoretical model because of the crucial importance of renegotiation, which in our view cannot be adequately formalized yet is crucial to take into account.

In the model, we treat exit clauses as a so-called "real options". The theory of options was initially developed to study financial instruments such as put options, which give the holder a right to sell a stock at a pre-determined price in the future. Once developed, this theory has also been leveraged to study other optional decisions, such as the option to acquire a Joint Venture (Kogut, 1991). Such applications of options theory outside of financial applications are called "real options theory". The main technique used is dynamic stochastic programming, which requires the use of stochastic differential equations (Øksendal, 1991;

Stokey, 2008). Our model builds off Dixit (1989) and Dixit & Pindyck (1994), who study firms' decisions to enter and exit markets in environments with fluctuations in output prices.

Our methodological contribution to real option theory is twofold. First, we extend the basic model of exit by a firm from a market to exit by a member from a political union. Unlike in a market, exit by one member of a political union implies the loss of benefits for the other members. By modeling this impact, we derive conditions for socially efficient exit in the setting of political unions. Second, we formalize the mathematics of state-contingent exit penalties: penalties which depend on the state of the other member in a union.

In what follows, we assume for simplicity that exit is definitive and that there is no possibility of re-entering later. However, even if re-entry is free the qualitative conclusions from our model would still hold, but the solution process would be more complicated as the optimal exit decisions and optimal entry decisions would be mutually interdependent.

In the decision-theoretical version of the model, we assume that only one member can exit from the agreement, and only consider that member. Hence for now we drop the index i to lighten notation. When thinking about exit in a continuous time set-up, a member compares the expected value of maintaining the agreement to the value of exiting right now. Assuming rational behavior, the expected value of maintaining the agreement should reflect optimal exit behavior in the future. This is captured by the notion of continuation value: the continuation value $V(x)$ is the expected discounted benefit from maintaining the agreement when the current state is x, assuming optimal exit behavior in the future (Dixit & Pindyck, 1994).

We show in the Appendix that the continuation value is composed of two parts: the expected perpetuity value from maintaining the union forever, plus the option value of terminating the union. At state x, the benefit flow is $x + \theta$. Since on average the future value of a Brownian Motion is equal to the current value, the expected perpetuity value at state x is $\frac{x+\theta}{r}$. The more important the future, the lower the discount rate r and the higher the perpetuity value. As we show in the Appendix, the option value of terminating the union

depends on the exit terms. Its general value is $Be^{\beta x}$, with $\beta = -\sqrt{\frac{2r}{\sigma^2}}$ and B a constant to be determined. Combining the expected perpetuity value and the option value, the value function can be written as

$$V(x) = \frac{x+\theta}{r} + Be^{\beta x}, \qquad \beta = -\sqrt{\frac{2r}{\sigma^2}} \tag{7}$$

Optimal exit with an exit penalty c

The continuation value is the value of maintaining the agreement for now, assuming optimal behavior for the future. Hence the constant B will depend on the optimal exit state x^e. In the Appendix, we show that when facing an exit penalty c, the optimal exit state and the corresponding value for B are

$$x^e = -\theta - r(k+c) + \frac{1}{\beta}, \qquad B = -\frac{e^{-\beta x^e}}{r\beta} \tag{8}$$

The optimal exit state consists of three components. The first one, $-\theta$, is the most intuitive. When x reaches $-\theta$, the benefit flow drops to zero: $\pi(t|x(t) = -\theta) = -\theta + \theta = 0$. The higher a member's type θ, the more negative the optimal exit state, i.e. the longer it is optimal to stay in the agreement: $\frac{\partial x^e}{\partial \theta} < 0$. The second term, $-r(k+c)$, reflects the deterring effect of exit costs since $\frac{\partial x^e}{\partial k} = \frac{\partial x^e}{\partial c} < 0$. The benefit flow needs to drop to $-r(k+c)$ to make exit worth considering, since the corresponding expected perpetuity value at that state would be equal to the exit costs $-(k+c)$.

The third term, $\frac{1}{\beta}$, is the least intuitive but can be interpreted as the optimal forbearance level. It reflects sophisticated rational behavior: given that re-entry is precluded, one should be willing to sustain some losses in the hope that the state improves again.[6] Intuitively, it seems clear that the higher the variance of the benefits, the higher the possibility that a bad state would improve substantially again, and the more reluctant one should be to exit. This intuition is confirmed: since $\frac{1}{\beta} = -\sqrt{\frac{\sigma^2}{2r}}$, it is easy to show that $\frac{\partial x^e}{\partial \sigma} < 0$.

[6] In fact, even if re-entry is free one should be willing to sustain some losses before exiting (Dixit, 1989).

Given (7) and (8), the continuation value at state x is

$$V(x) = \frac{x+\theta}{r} - \frac{e^{\beta(x-x^e)}}{r\beta} \quad (9)$$

The continuation value consists of the perpetuity value plus the option value of optimal exit. As the state deteriorates to the optimal exit state, the continuation value converges to the cost of exit: $\lim_{x \to x^e} V(x) = -(k+c)$. The higher the cost of exit, the lower the value of the exit option and hence the continuation value: $\frac{\partial V}{\partial k} = \frac{\partial V}{\partial c} < 0$. Members prefer for themselves an exit penalty of $c_i = 0$.

Enforcement problems: the possibility of a secession war

Up to now, we have assumed the exit clause was fully binding. A member could only exit by paying the ex-ante agreed exit penalty c. However, given the nature of currency unions and (con)federations, which cannot be enforced by supranational courts, the agreement may not be fully binding. In particular, the members may decide to renegotiate, or a member wishing to exit may do so unilaterally.

By "secession war", we denote any unilateral exit which does not respect a pre-agreed exit clause and has not been negotiated (if there is no exit clause). Hence our use of the term also includes figurative secession wars. We model a secession war as a pair of costs (w, d), where w is the cost the exiting member would have to incur to win the secession war (including both the direct costs and the reputational costs) and d is the corresponding damage to the remaining member.

The direct costs of a secession war will depend crucially on the type of agreement: getting out of a currency union can be done much more easily than out of a federation. Even among currency unions and federations large variations can be expected depending on the set-up and context. Currency unions with a common currency issued by one central bank will be much harder to get out of than currency unions based on freely interchangeable but separate currencies. Similarly, overseas regions will incur less costs in seceding unilaterally than regions which are fully contained within a federation.

The reputational costs of a secession war depend both on the international norm against unilateral exit and on the circumstances. For instance, regions with oppressed ethnic minorities will generally incur low

reputational costs, as they will be quickly welcomed by the international community (Buchanan, 1997; Sorens, 2016). Barring specific geopolitical concerns, they tend to be quickly recognized by the UN and the WTO. Rich regions wishing to escape from fiscal transfers will not be treated so favorably.

Clearly, if a member has the choice between using an exit clause and engaging in a secession war, it will pick the option with the lowest cost. Hence exit clauses with costs $c + k$ above w will be ineffective as they cannot be enforced: the exit clause will never be used, but a secession war will occur if the state drops below $x^e(w) = -\theta - rw + \frac{1}{\beta}$ and a negotiated exit is not forthcoming. The higher the cost w, the less relevant the secession war option and hence the lower the enforcement problem.

The consequences of exit for the other member of the union

Until now, we have only considered the member with exit option and dropped the index i from all formulas. We now reintroduce the index i for the member with the exit option, and introduce the index j for the member subject to potential exit by member i. For j, the value $V_j(x_i, x_j)$ of being in the agreement depends on both x_i and x_j. This is because j's benefit flow depends on x_j, but will be stopped by i depending on x_i.

In the Appendix, we show that if only i has an exit option, j's value of being in the agreement is given by

$$V_j(x_i, x_j) = \frac{x_j + \theta_j}{r} + \left(c_i - k_j - \frac{x_j + \theta_j}{r}\right) e^{\beta_i(x_i - x_i^e)} \qquad (10)$$

Given the initial state of the world $x_i = x_j = 0$, the ex-ante expected value for j is $V_j(0,0) = \frac{\theta_j}{r} + \left(c_i - k_j - \frac{\theta_j}{r}\right) e^{-\beta_i x_i^e}$. The first order condition corresponding to the optimal exit penalty from j's perspective is $\left(1 + r\beta_i\left(c_i - k_j - \frac{\theta_j}{r}\right)\right) e^{-\beta_i x_i^e} = 0$. It is easy to verify that this penalty is $c_i = \frac{\theta_j}{r} + k_j - \frac{1}{r\beta_i}$ and that it indeed corresponds to a maximum. This is the exit penalty preferred by j for the other member i. If only i has an exit option, then j would prefer for i's exit penalty to be higher the higher j's expected

benefits, i.e. the higher j's type θ_j. Similarly, the higher j's one-off costs, the higher j would like i's exit penalty to be.

The possibility of an anti-secession war

We have argued that agreements may not be fully binding because of the possibility of a secession war and of a negotiated exit. A further enforcement problem is the possibility of an anti-secession war. Even if the agreement includes an exit clause, a member trying to make legal use of this clause may be faced with an anti-secession war by the remaining member. This is what occurred during the break-up of the Soviet Union, which tried preventing the secession of some Soviet republics. In the case of the American Civil War, some Southerners claimed there was an implicit secession right and that hence the North was conducting an illegal anti-secession war (Buchanan, 1997: 36).

The risk of an anti-secession war is higher the higher the state of the remaining member and the lower the exit cost, and hence poses a lower limit on enforceable exit costs. Similar to a secession war, an anti-secession war can be modeled as a pair of costs: (1) the cost the remaining member would have to incur to prevent secession by the other member (including both the direct costs and the reputational costs) and (2) the corresponding damage to the member which tried to secede legally but failed. The stronger the international norm against preventing legal secession, the higher the reputational cost of trying to prevent legal secession and hence the less relevant the anti-secession war option as an enforcement problem. As democracies are more sensitive to reputational costs, such costs can conceivably account for the higher occurrence of anti-secession wars in the breakup of non-democratic states such as the Soviet Union.

Socially efficient exit

In this section we study exit from the point of the entire union. In a union with two members, exit by one member leads to the end of the union. By deriving the impact of exit on the combined benefits of the two players x_C, we derive a condition for socially efficient exit. Taking the benefits of both members into account, this conditions stipulates in which states of the world (x_i, x_j) the union should be preserved, and

in which states it should be terminated. To ease the exposition, we present the derivation of the socially efficient exit state as stemming from a social planner who has as an objective function the sum of the members' benefit flows.

We start by showing that if the benefit flows of both members are Brownian Motions, then their combined benefit flow is also a Brownian Motion. Recalling that the increments ϵ_{At} and ϵ_{Bt} of the Brownian motions $x_A(t)$ and $x_B(t)$ are jointly normally distributed for all t, the combined benefit flow π_C can be written as

$$\pi_C(t) = \pi_A(t) + \pi_B(t) = x_C(t) + \theta_C \tag{11}$$

where the combined state $x_C(t) = x_A(t) + x_B(t)$ and the combined type $\theta_C = \theta_A + \theta_B$. Using the definition of x_C, the increments $dx_C = \sigma_A \epsilon_{At}\sqrt{dt} + \sigma_B \epsilon_{Bt}\sqrt{dt}$ are normally distributed with variance $\sigma_C^2 = \sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B$. The variance of the combined state is determined by the variances of the states of the members, and by their correlation ρ.

The social planner's continuation value is defined in terms of the Brownian motion $x_C(t)$. We show in the Appendix that the condition for socially efficient exit can be expressed in terms of x_A and x_B:

$$x_C = x_C^e \Leftrightarrow x_A + x_B = -(\theta_A + \theta_B) - rk_C + \frac{1}{\beta_C} \tag{12}$$

Socially efficient exit requires taking into account the states of both members, as well as the total one-off costs of separation of $k_C = k_i + k_j$. If the combined state $x_A + x_B$ drops below the threshold specified in (35), terminating the union is socially efficient. Consistent with the notion of efficient breach presented in the introduction, socially efficient exit may require hurting one member if this benefits the other member more. If one member, say A, is in a good state (a high x_A), while B is in a bad state (a low x_B), the socially efficient exit decision will be taken by adding up both states and comparing the result with the total one-off costs of the separation $k_A + k_B$.

From the social planner's point of view, exit penalties are internal transfers and do not matter. However, as we have shown in (23) and (32) for the individual members exit penalties do matter. The higher the exit

penalty c_i, the lower the state x_i^e at which member i will exit. In the next section, we show that the exit penalty c_i can be defined so as to induce i to make socially efficient exit decisions.

State-contingent exit penalties

In the Appendix, we derive i's optimal exit behavior if only i has an exit option but the penalty can be made state-contingent. We show that if the penalty is of the form $c_i(x_j) = ax_j + b$, then i will exit in states (x_i^e, x_j^e) satisfying

$$x_i^e + arx_j^e = -\theta_i - r(b + k_i) + \frac{1}{\lambda_i}, \qquad \lambda_i = -\sqrt{\frac{2r}{\sigma_i^2 + a^2r^2\sigma_j^2 + 2ar\rho\sigma_i\sigma_j}} \tag{13}$$

For this condition to lead to socially efficient exit as in (12), one finds $a = \frac{1}{r}$, so that $\lambda_i = \beta_i$ and $b = \frac{\theta_j}{r} + k_j$. Hence the socially efficient exit penalty for i is

$$c_i^* = \frac{x_j + \theta_j}{r} + k_j \tag{14}$$

The higher the state of the other member x_j, the higher the exit cost should be to induce member i to take socially efficient exit decisions. This is intuitive: the more member j is enjoying the union, the higher should be i's penalty for ending the union. Conversely, if member j is in a very bad state, member i would receive a positive payment for ending the agreement. The second term of c_i^*, equal to k_j, reflects that for socially efficient exit i needs to be incentivized to take j's one-off costs of i's exit into account.

Under the socially efficient exit penalty c_i^*, i's option value coincides with the social planner's option value:

$$c_i = c_i^* \Rightarrow V_i(x_i, x_j) = \frac{x_i + \theta_i}{r} - \frac{e^{\beta_C(x_c - x_C^e)}}{r\beta}, \qquad V_j(x_i, x_j) = \frac{x_j + \theta_j}{r} \tag{15}$$

This is because i has the same costs and benefits from stopping the agreement as the social planner: the payment of $k_i + k_j$, and the loss of the perpetuity value $\frac{x_i + x_i}{r}$. The loss of x_i/r is direct: by stopping the agreement, i stops his benefit flow. The loss of x_j/r is indirect: the socially efficient exit penalty requires that i pay x_j/r to j.

Optimal exit in a game-theoretical setting

In the previous section, we formally analyzed the case where only one member had an exit option. In this section, we discuss exit in a game-theoretical setting. If both members have an exit option, member i's optimal exit strategy will depend on member j's exit strategy and vice versa. In such a setting, no analytical solutions are available because the exit options are no longer perpetual: member i's (stochastic) exit time is the expiration date of member j's exit option. In addition, the analysis in this setting depends crucially on the possibility of a negotiated exit.

If there are no obstacles to negotiating exit, socially efficient exit will occur irrespective of a potential exit clause. However, since the exit clause represents an outside option it may affect the side-payments made to forestall or obtain exit, similar to the distribution of property rights in Coase (1960). Imagine an agreement with an exit clause, and suppose member i is in a low state while member j is in a high state. If the exit cost is below the level that would induce socially efficient exit by i, member j will make a side-payment to member i for not exercising its exit option. The lower the exit cost, the higher this side-payment will need to be. Conversely, if the exit cost is above the level that would induce socially efficient exit by i, member i will have to make a side-payment to member j relative to the socially efficient exit penalty in order to be allowed to exit at the socially efficient state.

If the negotiation of exit is frictionless, secession wars will never occur, although their possibility may affect the side-payments made for socially efficient exit. However, in reality many obstacles may limit the scope for negotiating exit. Politicians or voters may be loss-averse or boundedly rational (Herweg & Schmidt, 2015). Moreover, transfers may be costly and members' states may be privately observed (Bordignon & Brusco, 2001). The importance of such impediments to a negotiated exit is ultimately an empirical matter. But because these impediments are hard to measure, we prefer not to formally model the role of negotiated exit and provide a discussion instead.

If negotiating exit is not frictionless, the possibility of an inefficient secession war reemerges, and with it the scope for exit clauses to improve social welfare. In particular, setting the exit cost $c_i + k_i$ equal to the

cost of a secession war w_i would be welfare enhancing: when i's state drops to $x_i^e(w_i)$, it will pay the penalty $c_i = w_i - k_i$ to j, instead of starting a secession war which costs d_j to j. If the state of the world is fully contractible, efficiency could be further increased in the absence of renegotiation by making member i's exit cost conditional on j's state as in (15). However, since the lack of observability of j's state by i may precisely be one of the impediments to a negotiated exit, it seems optimistic to assume that state-contingent exit clauses could fully restore social efficiency.

To conclude this section we summarize the predictions of our theory based on the decision theoretical model and our discussion of negotiated exit. We identified three potential modes of exit: ex-post negotiation, unilateral exit, and exit based on an ex-ante agreed exit clause. If both members of a union are benefitting from it (are in a good state), neither will consider exit. Hence in states of the world (x_i, x_j) where both x_i and x_j are high the union continues. If both i and j are in a bad state, i.e. x_i and x_j are both low, exit will be negotiated and the union is dissolved.

If i is in a bad state (low x_i) while j is in a good state (high x_j), i's decision whether to exit will depend on the exit costs of his cheapest mode of exit. If there is an exit clause, the associated exit costs consist of the penalty c_i and the one-off costs k_i. The costs of unilateral exit (secession war) are w. The higher i's exit costs and the lower the variance σ_i, the lower the state x_i can drop until exit becomes optimal for i. If there are no barriers to ex-post negotiation, exit will occur when it is socially efficient, i.e. when the combined benefits of both members drop below a critical level. The side-payment made for such a negotiated exit will depend on i's exit costs, and on j's costs of an anti-secession war. If i faces low exit costs and j high anti-secession costs, j will make a side-payment to i for staying in the agreement.

If a negotiated exit is not possible because dissolving the union would not be efficient (i.e. j is benefitting more from the union than i is losing), or because of barriers to negotiation, then i will consider exit based on an exit clause or unilateral exit.

The breakup of Yugoslavia

In this section we illustrate how our model can be applied to the breakup of Yugoslavia. We first give an overview of the events and discuss existing explanations in the literature. Next we illustrate the strengths and shortcomings of our theory in explaining the events.

From the 1946 creation of Yugoslavia to the 2013 Brussels agreement

After World War II, Tito united Yugoslavia as a Socialist Federal Republic consisting of six republics: Slovenia, Croatia, Bosnia-Herzegovina, Serbia, Montenegro, and Macedonia (Lampe, 2000: 233-235). The territory of Serbia contained two autonomous regions: Kosovo and Vojvodina. In 1974, a confederal constitution was adopted which gave extensive autonomy to each of the six republics and the two autonomous regions, and essentially gave all eight of them a veto at the federal level (Fine, 2003: 182). A map of the former Yugoslavia with currently recognized international boundaries is shown in Figure 1.

Figure 1. Former Yugoslavia: Wikimedia Commons map based on the Cartographic Section of the United Nations.

In 1980 Tito died. In the 1980s the Yugoslavian economy deteriorated rapidly (Shoup, 2008: 333), and a foreign debt crisis developed (Lampe, 2000: 300). In June 1991 Slovenia and Croatia unilaterally declared

their independence, and by the end of the year Macedonia did the same (Hupchick & Cox, 2001: 49). Serbia, controlled by Milošević, was opposed against these unilateral secessions, at least in part because of the Serbian minorities present in these other Yugoslav republics. In Slovenia, fighting only lasted for ten days and independence was quickly conceded. In Croatia, more fighting took place but the rump of Yugoslavia ultimately conceded independence. There was no resistance against Macedonia's independence. In Bosnia-Herzegovina, a three-way war erupted between Bosniaks, Croats and Serbs. Bosnia-Herzegovina declared its independence in March 1992. Slovenia, Croatia and Bosnia-Herzegovina were admitted as member states of the United Nations on 22 May 1992. However, the war in Bosnia-Herzegovina continued.

The two remaining republics in rump Yugoslavia, Montenegro and Serbia, proclaimed the Federal Republic of Yugoslavia in April 1992. In 1995 the Dayton agreement was signed to restore peace in Bosnia-Herzegovina, and in 1996 the Federal Republic of Yugoslavia recognized Bosnia-Herzegovina. In 2003, the Federal Republic of Yugoslavia renamed itself the State Union of Serbia and Montenegro, and adopted a new constitution. Article 60 of the new constitution stipulated that either state could declare independence after a three year waiting period. In 2006, Montenegro declared its independence after a referendum. Montenegro initially proposed a simple majority pro-independence threshold. However, under pressure from the EU they accepted a 55% threshold. The outcome of the referendum on 21 May 2006 was 55.5% in favor of independence, which Montenegro then declared. It was admitted to the UN in June 2006 as its 192th member state.

In the Serbian autonomous region Kosovo, tensions between Albanian nationalists and Serbia increased throughout the 1990s (Hupchick & Cox, 2001: 50). In 1998, the Kosovë Liberation Army (KLA) instigated an uprising, which led to further repression of Kosovar Albanians by Serbia. Atrocities committed by the Yugoslav National Army (JNA) increased, and NATO launched airstrikes on Serbia until Milošević conceded defeat in June 1999. In February 2008, Kosovo officially proclaimed its independence. In an advisory opinion, the International Court of Justice found that Kosovo's declaration of independence did not violate international law (ICJ, 2010). Since 2008, Kosovo has been recognized by over 100 UN member states. Serbia has not officially recognized Kosovo, but in 2013 the prime ministers of Serbia and Kosovo

accepted to normalize relations in the so-called Brussels Agreement mediated by EU High Representative Catherine Ashton.

Existing explanations for the dissolution of Yugoslavia

Before showing how our model can shed light on the dissolution of Yugoslavia, it is appropriate to first list explanations given in the literature. In her review of the subject, Dragovic-Soso (2008) lists five categories of prevailing explanations. The first category groups the arguments centered on deep-seated ethnic animosities, the so-called "ancient hatreds" often emphasized in popular accounts and in Western policy circles at the time. The second category focuses on ideology, typically contrasting Tito's federal Yugoslav ideology with conflicting Greater Serbia unitarian ideology and Croatian nationalist ideology. The third category stresses institutional legacies, focusing either on the dynamic of republican competition created by the 1974 confederal constitution, or on economic failure due to the socialist legacy. The fourth category emphasizes political and intellectual agency, i.e. the role of powerful individuals such as Milošević in Serbia, Tudjman in Croatia or Kučan in Slovenia. Finally, the fifth category groups arguments centered on international politics, such as German support for Slovenia and Croatia's independence, and the perception in the richer republics that EU accession would be easier for them outside of Yugoslavia.

Strenghts of our theory in explaining the events

Clearly, there is no shortage of explanations for the events in former Yugoslavia. We now highlight how our theory can provide a framework for understanding the dissolution of Yugoslavia and then discuss its limitations. We analyze all six cases of independence declarations in Yugoslavia: Slovenia (June 1991), Croatia (June 1991), Macedonia (September 1991), Bosnia-Herzegovina (March 1992), Montenegro (June 2006), Kosovo (February 2008). The advantage of analyzing the breakup of Yugoslavia is that many of the parameters were the same for each of the six cases. For instance, the constitution of the Socialist Federal Republic of Yugoslavia had no exit clauses. This constitution was applicable to the independence declarations of Slovenia, Croatia, Macedonia and Bosnia-Herzegovina.

Our theory centers on three key variables: (1) the net benefits of being in the union versus outside, (2) the terms of a potential exit clause, (3) the costs of unilateral secession and anti-secession wars. To demonstrate the explanatory power of the theory, we will show how similarities and differences in parameters affecting these three variables can explain the order and the nature of the independence of the republics and Kosovo.

As stated before, the net benefits of being in a federal union consist of economies of scale in the provision of public goods, the internalization of externalities, net incoming fiscal transfers, minus welfare losses from centralized decision-making in the presence of heterogeneity across the federated members (Alesina & Spolaore, 2003; Desmet et al., 2011). Two key external parameters can decrease the benefits from economies of scale: free trade and peace. First, with more free trade outside of the union the economies of scale from having a large internal market become less important. Second, the lower external military threats, the less important the economies of scale in the provision of national defense.

The parameters affecting the net benefits enumerated above clearly played a role in Yugoslavia. Croatia and Slovenia, the first republics to declare independence, were economically more advanced than the other republics (Lampe, 2000: 336) and were paying transfers to the rest of Yugoslavia (Fine, 2003: 182). One key mechanism for such transfers was the Yugoslav Federal Fund, which provided investment capital for the less developed regions (Bookman, 1993: 97). Hence in terms of fiscal transfers Croatia and Slovenia were on the losing side of the federation. They also suffered welfare losses from central decisions imposed on them by Belgrade - or at least they posed as such (Fine, 2003: 179). One concrete example of such a welfare loss is the systematic preference given to Serbian variants in the Serbo-Croation dictionary (Lampe, 2000: 305). Once the rich republics Croatia and Slovenia had seceded, the transfers they provided to the poorer regions stopped. Consistent with our theory, this may explain why it became beneficial for the poorer regions of Macedonia and Bosnia-Herzegovina to secede after Croatia and Slovenia had left the federation.

It is worth repeating that in our model, the benefits of the union are defined versus the outside option. The attractiveness of the outside option may be different for the different members of the union and may vary over time. With the adoption of the Single European Act in 1986, the European single market became

a tempting outside option for the more developed republics Croatia and Slovenia. Slovenia joined the EU in 2004 and Croatia in 2013. Macedonia, Montenegro and Serbia are currently candidate Member States of the EU, while Bosnia-Herzegovina and Kosovo are potential candidates. Note that the EU refers to Macedonia as the Former Yugoslav Republic of Macedonia due to Greek objections to the name "Macedonia".

In terms of economies of scale in defense spending, the disappearance of the external Soviet threat as of 1989 clearly decreased this advantage of being part of the Yugoslav federation (Cohen, 2008: 374). While this external factor may not be helpful in explaining the order of independence, it seems no coincidence that Yugoslavia only started disintegrating in 1991, after the disappearance of the Soviet threat. A related point is the possibility of NATO-accession outside of Yugoslavia. In part to benefit from aid and investment from both the capitalist and the Soviet bloc, Yugoslavia had a policy of non-alignment (Shoup, 2008: 336). Outside of Yugoslavia, independent republics have been free to seek membership of NATO and benefit from much larger economies of scale in defense. Slovenia joined NATO in 2004 and Croatia in 2009. In December 2015, Montenegro was officially invited to join NATO.

There was no exit clause available to Slovenia, Croatia, Bosnia-Herzegovina, Macedonia or Kosovo so they all became independent through a unilateral declaration. In 2006, Montenegro used the exit clause that was entered in the constitution of the State Union of Serbia and Montenegro in 2003. Montenegro respected the waiting period of three years and Serbia accepted the independence peacefully. The fact that the exit clause was called just shortly after the stipulated three-year waiting period is suggestive that exit clauses really can have a binding role in enabling peaceful secession.

In setting up our model, we argued that the cost of unilateral secession and anti-secession wars is determined by both tangible components (military expenses) and intangible components (reputation costs). If there is no exit clause, unilateral declarations of independence will occur if their cost is lower than the cost of staying in the federation. The tangible costs of unilateral secession and anti-secession war depend importantly on geographical factors and on the presence of standing armies. In this regard the fact that independence was declared first by outermost republics (Slovenia, Croatia and Macedonia) comports well

with our theory: the further from the center a federal member is, the more expensive it is to military prevent its secession. No troops of the Yugoslav National Army (JNA) were present in Slovenia at the time of the independence declaration in June 1991, giving the Slovenes time to prepare.

The reputation costs of unilateral secession and anti-secession wars depend crucially on the stance taken by third countries and organizations such as the UN concerning diplomatic recognition. Third countries and the UN tend to look more favorably upon so-called "remedial secession" by oppressed ethnic minorities, and in such cases to tend to be in favor of a right to self-determination (Buchanan, 1997; Dietrich, 2013; Sorens, 2016). Germany clearly played a role in lowering the reputation costs of unilateral secession by promising to recognize the independence of Croatia and Slovenia. Conversely, US policy played a role in lowering the cost of the Serbian anti-secession war in Kosovo. Indeed, in spite of condemning Serb violence, the US "insisted that Kosovo should remain within Serbia" and hence effectively gave Serbia "green light" for trying to keep Kosovo in Serbia militarily (Fine, 2003: 187).

To conclude, our theory provides a plausible account of reasons of Yugoslavia's breakup and the order of the independence declarations by Slovenia (June 1991), Croatia (June 1991), Macedonia (September 1991), Bosnia-Herzegovina (March 1992), Montenegro (June 2006), and Kosovo (February 2008). The breakup as a whole can be explained by lack of net benefits for each of Yugoslavia's constituent parts. Ethnic heterogeneity, with strong clustering along republic lines, meant that the republics were susceptible to welfare losses from centralized decision-making. The fact that the federation stayed intact until the 1990s suggests that the economies of scale of being in a larger country were still sufficient to compensate for the losses from heterogeneity. The end of the Cold War in 1989 and the increased attractiveness of joining the EU and NATO as an outside option meant that the benefits of being in Yugoslavia versus outside where substantially diminished as of 1989.

The fact that first the rich, transfer-paying republics of Slovenia and Croatia seceded is fully consistent with our cost-benefit centered model of secession. There was no constitutional exit clause available to these republics, but they faced limited costs of unilateral exit. They were located in a corner of the federation (making military intervention from the federation costly), and had received assurance from Germany and

others as to the recognition of their independence. This stands in sharp contrast with the case of Kosovo, which is located within Serbia, and about which the United States for a long time insisted that it should remain a part of Serbia. The orderly nature of Montenegro's independence, respecting the stipulated waiting period of three years, points to the beneficial role that exit clauses can have, as suggested by our theory.

Limitations of the empirical discussion on Yugoslavia and shortcomings of our theory

Clearly, the fact that our theory provides a plausible account of the events does not "prove" it. We merely claim that the facts are, in general, consistent with our theory. We leave for future work more systematic empirical research to further test and improve the theory, and conclude this empirical section by identifying three limitations of our theory. First, ethnicity clearly played a role in Yugoslavia beyond how it affected local preferences and the corresponding welfare losses from centralized policies.[7] Indeed, one could argue that decentralization should be able to mitigate these losses, and that such decentralization was present in Yugoslavia since the 1974 constitution.[8] Conversely, from the Serbian perspective, ethnicity clearly also played a role in its reaction to independence declarations. The most violent reactions occurred in Kosovo, Bosnia-Herzegovina, and Croatia – where significant Serb minorities of over 10 percent were present (Lampe, 2000: 337, 368). The independence declarations of Slovenia and Macedonia, with little or no Serb minorities, were accepted after a ten-day war and no war at all.

A second limitation of our theory is that while "ancient hatreds" may be provided too easily as a comprehensive explanation of the Yugoslav dissolution, history clearly did play a role. The strong reaction of the Serbs against Croation independence can at least in part be explained by past history. During the second World War, the Croation Ustaše regime collaborated with the Nazis and killed hundred thousands of Serbs (Fine, 2003: 181). When Croatia declared its independence in 1991, some of the national symbols

[7] For a discussion of the relationship between ethnicity, culture and preferences see Desmet, Ortuño-Ortin, & Wacziarg (2016). For a view that in the Balkans only states borders mapped along ethnic line scan be stable, see Stokes (2003).

[8] For a recent paper on the direction of causality between decentralization and demands for autonomy or independence, see Cederman, Hug, & Wucherpfennig (2015).

from the Ustaše era were reintroduced, and Croatia was declared "a state of Croats" (Fine, 2003: 184). This was worrying for the Serbian minority in Croatia and in the eyes of some observers qualifies Milošević's "Greater Serbia" policy as a reaction to Croatian actions rather than an ex-ante imperialist plan. A third limitation of our formal model is that it focuses on structural factors and hence leaves out the role of political agency. Politicians such as Milošević in Serbia and Tudjman in Croatia clearly stoked ethnic animosity for personal political gains, causing a general "ethnicization of politics" (Cohen, 2008).

Conclusion

We presented a political economic analysis of exit and exit clauses in currency unions and (con)federations. In our model, a member's benefits from a political union are determined by its type θ and its state x. Because of changes in the state of the world, a member's benefits may go up or down. Mathematically, we assumed that the state of each member follows a Brownian Motion. If a member's benefits become negative, i.e. it is in a low state, it may wish to exit. Using real option theory, we derived a member's optimal exit state. The higher a member's type, the higher exit costs, and the higher the variance of benefits, the lower the state can drop until it becomes optimal to exit.

When a member exits, the benefits for the other members of that member being in the union are also stopped. From a social efficiency perspective, exit should occur if and only if it generates value taking all members' benefits into account: this is the principle of "efficient breach" in contract law. In political unions, if exit costs are too low, a member may withdraw from a union even if that is harmful from a social perspective combining all members' benefits.

Exit costs depend on the mode of exit. If a political union has an ex-ante negotiated exit clause, exit can occur on the basis of such a clause. Exit clauses can contain several conditions, among which we analyzed exit penalties, one-off costs, and notice periods. If the costs of exit according to the exit clause are low, and it occurs while other members are benefitting a lot from the union, those other members may try to prevent exit forcefully. In practice, such prevention of legal exit will carry both military costs and reputational costs, especially for democracies. If there is no exit clause, or the exit cost is high, exit may also occur unilaterally.

The costs of such unilateral exit consist of tangible costs (such as military expenses) and reputational costs (which depend on the international recognition of the exit). A third mode of exit is through ex-post negotiation, although this can be expected to be difficult because of vested interests and frictions to negotiations.

Our findings run counter to the dominant point of view in the literature that exit clauses should be avoided in federations. The argument that exit clauses necessarily decrease the stability of an agreement is unsound: without exit clauses, exit may still occur unilaterally or through a negotiated exit. The inclusion of exit clauses when political unions are created can improve ex-ante social welfare. In particular, costly state-contingent exit penalties can enable socially efficient exit even if negotiating exit is difficult. If exit penalties cannot be made state-contingent, fixed exit penalties can still increase social welfare by avoiding secession wars.

Given our results, one may wonder why exit clauses are not more prevalent in real-world political unions. We discuss four main reasons here. First, the politicians negotiating political unions may wish to tie their successors' hands. In our model, we assumed that each member of the political union could be represented as a unitary actor. In practice, political unions may have heterogeneous effects on different parts of the population within one member. If that is the case, politicians whose electorate favors the agreement may want to prevent exit in the future by not having an exit clause.

Second, political unions may not be rationally designed. This may be especially true for federal countries which have existed for a long time. But even for modern federations and currency unions there may be a norm against exit clauses because they are perceived as going against the spirit of collaboration embedded in such unions. Such a norm may also explain why most real-world examples of exit clauses (such as the EU's Article 50) have no exit penalties, in spite of the necessity of exit costs to achieve socially efficient exit decisions.

Third, exit clauses may not be prevalent in the real world because some of the conditions of our theory do not hold. In particular, the cost of a secession war may be uncertain. Fourth, powerful members may be able to impose their terms upon the other members. Given favorable substantive terms of the union, they

would expect to benefit a lot from the agreement. In turn, this would lead them to prefer a high exit cost for the other members. Expecting to be able to renegotiate easily if the agreement would ever stop being beneficial to them, they may prefer not having an exit clause altogether, and impose this upon the other members. Empirical research is needed to assess the true role of each of the four reasons mentioned above.

We leave for future research the negotiation of exit clauses. Given the distributional implications of exit clauses, the bargaining process will be important for the final outcome, and socially efficient exit clauses are not guaranteed. However, to the extent that ex-ante information is limited and the members are negotiating behind a veil of ignorance, their ex-ante preferences will be closer to the social optimum and negotiations will be easier. A second area for future research is to make the link between the state of the world and payoffs endogenous. In this article, we have modeled the payoff-relevant state of the world as fully exogenous. In practice the members may be able to affect how the state of the world affects their payoffs by renegotiating the substantive terms of the agreement. For instance, a federal country's laws affect how the state of the world maps to interregional transfers and hence to the benefits from the agreement. If at some point the prevailing transfers make the federation undesirable to a member, this member will likely try to renegotiate the federal laws before considering exit from the federation.

References

Aghion, P., & Bolton, P. (2003). Incomplete social contracts. *Journal of the European Economic Association*, *1*(1), 38–67.

Alesina, A., & Spolaore, E. (2003). *The Size of Nations*. Cambridge, Mass: MIT Press.

Athanassiou, P. (2009). Withdrawal and expulsion from the EU and EMU: some reflections. *ECB Legal Working Paper Series*, (10).

Bookman, M. Z. (1993). *The economics of secession*. New York: St. Martin's Press.

Bordignon, M., & Brusco, S. (2001). Optimal secession rules. *European Economic Review*, *45*(10), 1811–1834.

Buchanan, A. (1997). Theories of Secession. *Philosophy & Public Affairs*, *26*(1), 31–61.

Cederman, L., Hug, S., & Wucherpfennig, J. (2015). Autonomy , Secession and Conflict : A Strategic Model. *ENCoRe Conference*, *October*, 1–50.

Chen, Y., & Ordeshook, P. C. (1994). Constitutional secession clauses. *Constitutional Political Economy*, *5*(1), 45–60.

Coase, R. (1960). The Problem of Social Cost. *Journal of Law and Economics*, *3*, 1–47.

Cohen, B. J. (1993). Beyond EMU: the Problem of Sustainability. *Economics & Politics*, *5*(2), 187–203.

Cohen, L. J. (2008). Disintegrative Synergies and the Dissolution of Socialist Federations: Yugoslavia in Comparative Perspective. In L. J. Cohen & J. Dragovic-Soso (Eds.), *State Collapse in South-Eastern Europe: New Perspectives on Yugoslavia's Disintegration* (pp. 365–396). West Lafayette, Indiana: Purdue University Press.

Desmet, K., Breton, M. Le, Ortuño-Ortin, I., & Weber, S. (2011). The stability and breakup of nations: a quantitative analysis. *Journal of Economic Growth*, *16*, 183–213.

Desmet, K., Ortuño-Ortin, I., & Wacziarg, R. (2016). Culture, Ethnicity, and Diversity. *Working Paper*, *May 2016*, 1–39.

Dietrich, F. (2013). Secession of the rich: A qualified defence. *Politics, Philosophy & Economics*, *13*(1), 62–81.

Dixit, A. K. (1989). Entry and Exit Decisions under Uncertainty. *Journal of Political Economy*, *97*(3), 620.

Dixit, A. K., & Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton, N.J: Princeton University Press.

Dragovic-Soso, J. (2008). Why did Yugoslavia Disintegrate? An Overview of Contending Explanations. In L. J. Cohen & J. Dragovic-Soso (Eds.), *State Collapse in South-Eastern Europe: New Perspectives on Yugoslavia's Disintegration* (pp. 1–39). West Lafayette, Indiana: Purdue University Press.

Drèze, J., De Grauwe, P., & Edwards, J. (1993). Regions of Europe: a Feasible Status, to be discussed. *Economic Policy*, *8*(17), 265–307.

Filippov, M., Ordeshook, P., & Shvetsova, O. (2004). *Designing Federalism: A Theory of Self-Sustainable Federal Institutions*. Cambridge University Press.

Fine, J. (2003). Heretical Thoughts about the Postcommunist Transition in Once and Future Yugoslavia. In N. Naimark & H. Case (Eds.), *Yugoslavia and its Historians: Understanding the Balkan Wars of the 1990s* (pp. 179–192). Stanford University Press.

Goetz, C. J., & Scott, R. E. (1977). Liquidated Damages, Penalties and the Just Compensation Principle: Some Notes on an Enforcement Model and a Theory of Efficient Breach. *Columbia Law Review*, *77*(4), 554–594.

Habtu, A. (2005). Multiethnic Federalism in Ethiopia: A Study of the Secession Clause in the Constitution. *Publius: The Journal of Federalism*, *35*(2), 313–335.

Helfer, L. R. (2005). Exiting treaties. *Virginia Law Review*, *91*(7), 1579–1648.

Herweg, F., & Schmidt, K. M. (2015). Loss Aversion and Inefficient Renegotiation. *The Review of Economic Studies*, *82*(1), 297–332.

Hupchick, D. P., & Cox, H. E. (2001). *Palgrave concise historical atlas of the Balkans*. New York, NY: Palgrave.

ICJ. (2010). Accordance with International Law of the Unilateral Declaration of Independence in Respect of Kosovo. *Advisory Opinion, International Court of Justice Reports 2010*, 403–453.

Kogut, B. (1991). Joint Ventures and the Option to Expand and Acquire. *Management Science*, *37*(1), 19–33.

Koremenos, B. (2016). *The Continent of International Law: Explaining Agreement Design*. Cambridge University Press.

Lampe, J. R. (2000). *Yugoslavia as History: Twice there was a Country* (2nd ed.). Cambridge, UK: Cambridge University Press.

Mörters, P., & Peres, Y. (2010). *Brownian Motion*. Cambridge University Press.

Øksendal, B. (1991). *Stochastic Differential Equations*. New York: Springer.

Rainer, H. (2007). Should we write prenuptial contracts? *European Economic Review*, *51*(2), 337–363.

Shoup, P. (2008). The Disintegration of Yugoslavia and Western Foreign Policy in the 1980s. In L. J. Cohen & J. Dragovic-Soso (Eds.), *State Collapse in South-Eastern Europe: New Perspectives on*

Yugoslavia's Disintegration (pp. 333–364). West Lafayette, Indiana: Purdue University Press.

Sorens, J. (2016). Hearing: U.S. Policy Toward National Self-Determination Movements. *House Committee on Foreign Affairs, March 15*, 1–6.

Stokes, G. (2003). Solving the Wars of Yugoslav Succession. In N. Naimark & H. Case (Eds.), *Yugoslavia and its Historians: Understanding the Balkan Wars of the 1990s* (pp. 193–207). Stanford University Press.

Stokey, N. L. (2008). *The Economics of Inaction: Stochastic Control Models with Fixed Costs*. book, Princeton: Princeton University Press.

Sunstein, C. R. (1991). Constitutionalism and Secession. *The University of Chicago Law Review*, *58*(2), 633–670.

Sunstein, C. R. (2001). Should Constitutions Protect the Right to Secede? A Reply to Weinstock. *The Journal of Political Philosophy*, *9*(3), 350–355.

Weinstock, D. (2001). Constitutionalizing the Right to Secede. *The Journal of Political Philosophy*, *9*(2), 182–2003.

Young, R. (1994). How Do Peaceful Secessions Happen ? *Canadian Journal of Political Science / Revue Canadienne de Science Politique*, *27*(4), 773–792.

Appendix

Derivation of the continuation value

At any point in time, in order for a member to be willing to maintain the agreement for dt longer, the expected change in the continuation value combined with the benefit flow (1) over dt should add up to the normal return $rV(x)dt$:

$$E[dV] + (x + \theta)dt = rV(x)dt \tag{16}$$

Determining the continuation value is just like pricing a stock with expected appreciation $E[dV]$ and dividend flow $x + \theta$. By Ito's lemma, $dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial x}dx + \frac{1}{2}\frac{\partial^2 V}{\partial x^2}(dx)^2$; see Øksendal (1991) for the theory of stochastic differential equations and a discussion of Ito's lemma. The benefit flow does not depend on calendar time directly: $\frac{\partial V}{\partial t} = 0$. Together with (2), $E[dx] = 0$ and $E[(dx_i)^2] = \sigma_i^2 dt$, this implies

$$dV = V'(x)dx + \frac{1}{2}V''(x)\sigma^2\epsilon_t^2 dt \Rightarrow E[dV] = \frac{1}{2}V''(x)\sigma^2 dt \tag{17}$$

By substituting (17) in (16), we find

$$\frac{1}{2}\sigma^2 V''(x) - rV(x) = -x - \theta \tag{18}$$

This is a second order differential equation in x: we solve it by first identifying the solution $V_h(x)$ of the homogeneous equation $\frac{1}{2}\sigma^2 V''(x) - rV(x) = 0$. Next we identify a particular solution $V_p(x)$ which satisfies the equation $\frac{1}{2}\sigma^2 V''(x) - rV(x) = -x - \theta$. The general solution is given by the particular solution plus any linear combination of solutions to the homogeneous equation. For the homogeneous part, we try a solution of the form $V(x) = e^{\lambda x}$. This yields

$$V_h(x) = Ae^{\alpha x} + Be^{\beta x}, \qquad \alpha = \sqrt{\frac{2r}{\sigma^2}}, \beta = -\sqrt{\frac{2r}{\sigma^2}} \tag{19}$$

with constants A and B to be identified. Note that $\alpha > 0$ and $\beta = -\alpha < 0$. For a particular solution, we try $V(x) = ax + b$. This yields the solution $V_p(x) = \frac{x+\theta}{r}$. This is the expected perpetuity value from the

agreement starting from state x. Combining the particular solution with the homogeneous part, the general solution is

$$V(x) = V_p(x) + V_h(x) = \frac{x+\theta}{r} + Ae^{\alpha x} + Be^{\beta x} \quad (20)$$

Since $V_p(x)$ represents the value from maintaining the agreement perpetually, $V_h(x)$ represents the option value of exit, which should be positive. As the state improves, the value of the exit option should converge to 0: the better the state, the higher your benefit flow and the less valuable the exit option. This implies that $A = 0$ since $\alpha > 0$. We now have an expression for the continuation value $V(x)$ up to the constant B. This constant will be determined by a boundary condition corresponding to optimal exit.

Optimal exit with a penalty c

Two conditions are needed for optimal exit in a continuous time stochastic model (Dixit & Pindyck, 1994). The first is *Value Matching (VM)*: exit should occur when $V(x)$ drops to the value of the outside option. In our model, the outside option consists of paying the one-off costs and the penalty c, which corresponds to a value of $-k-c$. The second condition is *Smooth Pasting (SP)*: optimal stopping requires that $V'(x)$ be equal to the derivative of the value of being outside of the agreement – which is 0 in our case, since the exit value does not depend on the state. Grouping the two conditions we obtain a system with two equations and two unknowns (x^e, B):

$$V(x^e) = -k-c \Leftrightarrow \frac{x^e+\theta}{r} + Be^{\beta x^e} = -k-c \quad \textbf{\textit{VM}} \ (21)$$

$$V'(x^e) = 0 \Leftrightarrow \frac{1}{r} + \beta B e^{\beta x^e} = 0 \quad \textbf{\textit{SP}} \ (22)$$

The solutions for (x^e, B) are

$$x^e = -\theta - r(k+c) + \frac{1}{\beta}, \qquad B = -\frac{e^{-\beta x^e}}{r\beta} \quad (23)$$

Optimal exit with a notice period N

Exit clauses specifying an exit penalty c are not the only option. Another type of exit provision is a notice period: exit is free but needs to be announced N periods in advance. In this case, the expected value of exit at state x is equal to

$$V^N(x) = -k + \int_0^N (x+\theta)e^{-rt}dt = -k + \frac{1}{r}(1-e^{-rN})(x+\theta) \qquad (24)$$

By the same reasoning as (20), the continuation value is $V(x) = \frac{x+\theta}{r} + Be^{\beta x}$. The following value-matching and smooth-pasting conditions determine the constant B and the optimal exit state x^e:

$$V(x^e) = V^N(x^e) \Leftrightarrow \frac{x^e+\theta}{r} + Be^{\beta x^e} = -k + \frac{1}{r}(1-e^{-rN})(x+\theta) \qquad \textbf{\textit{VM}}\ (25)$$

$$V'(x^e) = [V^N]'(x^e) \Leftrightarrow \frac{1}{r} + \beta Be^{\beta x^e} = \frac{1}{r}(1-e^{-rN}) \qquad \textbf{\textit{SP}}\ (26)$$

Solving this system of two equations one finds:

$$x^e = -\theta - rk + \frac{1}{\beta}, \qquad B = -\frac{e^{-rN-\beta x^e}}{r\beta} \qquad (27)$$

This shows that a notice period does not deter exit: the optimal exit state is identical to the one with exit penalty $c = 0$. Of course, a notice period may have benefits which we have not modeled here, such as lowering the one-off separation costs k. The continuation value is $V(x) = \frac{x+\theta}{r} - \frac{e^{\beta(x-x^e)-rN}}{r\beta}$. The longer the notice period, the less valuable the exit option and hence the lower the continuation value: $\frac{\partial V}{\partial N} < 0$.

The value function of a member without exit option

Just like (16), over an infinitesimal period dt the expected change in value combined with the benefit flow should add up to the normal return $rV_i(x_i, x_j)dt$:

$$E[dV_j] + (x_j + \theta_j)dt = rV_j(x_i, x_j)dt \qquad (28)$$

Ito's lemma in two dimensions (Øksendal, 1991) gives

$$dV_j = \frac{\partial V_j}{\partial t}dt + \frac{\partial V_j}{\partial x_i}dx_i + \frac{\partial V_j}{\partial x_j}dx_j + \frac{1}{2}\left[\frac{\partial^2 V_j}{\partial x_i^2}(dx_i)^2 + \frac{\partial^2 V_j}{\partial x_j^2}\left(dx_j\right)^2 + 2\frac{\partial^2 V_j}{\partial x_i \partial x_j}dx_i dx_j\right] \quad (29)$$

Now, since V_j is time-independent and using the properties of the Brownian motions x_A and x_B, $E[dV_j] = \frac{1}{2}dt\left[\sigma_i^2\frac{\partial^2 V_j}{\partial x_i^2} + \sigma_j^2\frac{\partial^2 V_j}{\partial x_j^2} + 2\rho\sigma_i\sigma_j\frac{\partial^2 V_j}{\partial x_i \partial x_j}\right]$. Fill this out in (28) to obtain the partial differential equation

$$\frac{1}{2}\left[\sigma_i^2\frac{\partial^2 V_j}{\partial x_i^2} + \sigma_j^2\frac{\partial^2 V_j}{\partial x_j^2} + 2\rho\sigma_i\sigma_j\frac{\partial^2 V_j}{\partial x_i \partial x_j}\right] + x_j + \theta_j = rV_j(x_i, x_j) \quad (30)$$

If i exits, j receives the exit penalty c and incurs one-off cost k_j. Since i will exit at $x_i^e = -\theta_i - r(k_i + c_i) + \frac{1}{\beta_i}$, the appropriate value-matching condition is

$$\forall x_j\colon\ V_j\left(x_i^e, x_j\right) = c_i - k_j \quad \textbf{\textit{VM}}\ (31)$$

Since j undergoes i's exit decision, there is no smooth-pasting condition for optimality. The solution for $V_j(x_i, x_j)$ needs to satisfy both the partial differential equation (30) and the value-matching condition VM (31). The general solution of (30) consists of a particular solution and any linear combination of solutions to the homogeneous equation. A particular solution is again $V_{jp}(x_i, x_j) = \frac{x_j + \theta_j}{r}$, the perpetuity value of the agreement.

If the Brownian motions are uncorrelated and $\rho = 0$, then the partial differential equation is separable and analytical solutions can be obtained for the homogeneous equation as well. As before, one solution for the homogeneous equation is $e^{\beta_i x_i}$ with $\beta_i = -\sqrt{\frac{2r}{\sigma_i^2}}$. Since the particular solution $V_{jp} = \frac{x_j + \theta_j}{r}$ contains x_j but VM (31) does not, we should identify a solution for the homogeneous equation which also contains x_j. One can verify that $x_j e^{\beta_i x_i}$ is such a solution. Combining the particular solution with the two solutions of the homogeneous problem, we get $V_j\left(x_i, x_j\right) = \frac{x_j + \theta_j}{r} + Ce^{\beta_i x_i} + Dx_j e^{\beta_i x_i}$. Using VM (31), $C = \left(c_i - k_j - \frac{\theta_j}{r}\right)e^{-\beta_i x_i^e}$ and $D = -\frac{e^{-\beta_i x_i^e}}{r}$, so that

$$V_j(x_i, x_j) = \frac{x_j + \theta_j}{r} + \left(c_i - k_j - \frac{x_j + \theta_j}{r}\right) e^{\beta_i(x_i - x_i^e)} \quad (32)$$

Socially efficient exit

At the time of exit, the members incur the one-off costs k_i and k_j. From the social planner's point of view, any penalty c_i is a pure transfer from i to j and does not play a role when considering the union as a whole. Hence the social value of exit is $-k_C = -(k_i + k_j)$. The value-matching and smooth-pasting conditions are

$$V_C(x_C^e) = -2k \Leftrightarrow \frac{x_C^e + \theta_C}{r} + Be^{\beta_C x_C^e} = -k_C \quad \textbf{\textit{VM}} \ (33)$$

$$V_C'(x_C^e) = 0 \Leftrightarrow \frac{1}{r} + \beta_C B e^{\beta_C x_C^e} = 0 \quad \textbf{\textit{SP}} \ (34)$$

Resulting in the socially efficient exit state $x_C^e = -\theta_C - rk_C + \frac{1}{\beta_C}$ with $\beta_C = -\sqrt{\frac{2r}{\sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B}}$. The condition for socially efficient exit can be expressed in terms of the underlying states x_A and x_B:

$$x_C = x_C^e \Leftrightarrow x_A + x_B = -(\theta_A + \theta_B) - rk_C + \frac{1}{\beta_C} \quad (35)$$

State-contingent exit penalties

If the penalty c_i can be made state-contingent, then i's continuation value will depend on x_j. Assume only i has an exit option but the exit cost c_i is a function of x_j, specifically

$$c_i(x_j) = ax_j + b \quad (36)$$

As in (16), in order for i to stay in the agreement for dt longer at state (x_i, x_j), the expected change in continuation value combined with the benefit flow should add up to the normal return $rV_i(x_i, x_j)dt$

$$E[dV_i] + (x_i + \theta_i)dt = rV_i(x_i, x_j)dt \quad (37)$$

Ito's lemma in two dimensions gives

$$dV_i = \frac{\partial V_i}{\partial t}dt + \frac{\partial V_i}{\partial x_i}dx_i + \frac{\partial V_i}{\partial x_j}dx_j + \frac{1}{2}\left[\frac{\partial^2 V_i}{\partial x_i^2}(dx_i)^2 + \frac{\partial^2 V_i}{\partial x_j^2}\left(dx_j\right)^2 + 2\frac{\partial^2 V_i}{\partial x_i \partial x_j}dx_i dx_j\right] \quad (38)$$

Now, since V_i is time-independent and using the properties of the Brownian motions x_A and x_B, $E[dV] = \frac{1}{2}dt\left[\sigma_i^2\frac{\partial^2 V_i}{\partial x_i^2} + \sigma_j^2\frac{\partial^2 V_i}{\partial x_j^2} + 2\rho\sigma_i\sigma_j\frac{\partial^2 V_i}{\partial x_i \partial x_j}\right]$. Fill this out in (37) to obtain the partial differential equation

$$\frac{1}{2}\left[\sigma_i^2\frac{\partial^2 V_i}{\partial x_i^2} + \sigma_j^2\frac{\partial^2 V_i}{\partial x_j^2} + 2\rho\sigma_i\sigma_j\frac{\partial^2 V_i}{\partial x_i \partial x_j}\right] + x_i + \theta_i = rV_i(x_i, x_j) \quad (39)$$

As in the main text, a particular solution is $V_p(x_i, x_j) = \frac{x_i+\theta_i}{r}$, the perpetuity value of the agreement. For the homogeneous part, which corresponds to the option value of exit, try $V_h(x_i, x_j) = e^{\lambda_i x_i + \lambda_j x_j}$ to obtain

$$\sigma_i^2\lambda_i^2 + \sigma_j^2\lambda_j^2 + 2\rho\sigma_i\sigma_j\lambda_i\lambda_j = 2r \quad (40)$$

This gives $V_i(x_i, x_j) = \frac{x_i+\theta_i}{r} + Ce^{\lambda_i x_i + \lambda_j x_j}$ with C to be determined and λ_i, λ_j satisfying (40).

Since upon exit i has to pay the penalty $c(x_j) = ax_j + b$ and incurs the one-off cost k_i, the value-matching condition at an exit state $x^e = (x_i^e, x_j^e)$ is

$$V(x^e) = -c(x_j^e) - k_i \Leftrightarrow \frac{x_i^e + \theta_i}{r} + Ce^{\lambda_i x_i^e + \lambda_j x_j^e} = -c(x_j^e) - k_i \quad \textbf{\textit{VM}} \ (41)$$

There are two smooth-pasting conditions because i should take both x_i and x_j into account when thinking about exit. The penalty does not depend on x_i, but it does depend on x_j, so that

$$\frac{\partial V_i}{\partial x_i}(x^e) = 0 \Leftrightarrow \frac{1}{r} + C\lambda_i e^{\lambda_i x_i^e + \lambda_j x_j^e} = 0$$
$$\frac{\partial V_i}{\partial x_j}(x^e) = -\frac{\partial c}{\partial x_j} \Leftrightarrow C\lambda_j e^{\lambda_i x_i^e + \lambda_j x_j^e} = -a \quad \textbf{\textit{SP}} \ (42)$$

Combining conditions (40), ***VM*** (41) and ***SP*** (42), one finds

$$\lambda_i = -\sqrt{\frac{2r}{\sigma_i^2 + a^2r^2\sigma_j^2 + 2ar\rho\sigma_i\sigma_j}}, \qquad \lambda_j = ar\lambda_i \quad (43)$$

Filling this out, one finds that optimal exit for i is defined in terms of the following linear combination of x_i and x_j

$$x_i^e + arx_j^e = -\theta_i - r(b + k_i) + \frac{1}{\lambda_i} \quad (44)$$

The higher a or the more c_i is made contingent on x_j, the higher the weight of x_j in i's exit decision. Member i's continuation value depends on its own state x_i for the perpetuity value, and on the linear combination of x_i and x_j for the option value

$$V_i(x_i, x_j) = \frac{x_i + \theta_i}{r} - \frac{e^{\lambda_i\left[(x_i + arx_j) - (x_i^e + arx_j^e)\right]}}{r\lambda_i} \quad (45)$$

It is easy to verify that with a non-state contingent exit penalty $c_i(x_j) = c$, i.e. $a = 0, b = c$, the solution reduces to non-state contingent solution presented in the main text.