Limits of the WTO as a self-enforcing institution*

T. Renee Bowen†

November 22, 2010

Abstract

Is there a limit to trade cooperation that the WTO can facilitate? In this paper I present a theory of the WTO in which the WTO is an equilibrium outcome of multiple bilateral repeated prisoners' dilemma games among countries. The equilibrium mimics an important feature of the WTO’s Dispute Settlement Mechanism (DSM) - trading partners withhold retaliation during the DSM process. I call this property of the DSM forbearance. I show that when a sufficient number of countries participate in multilateral sanctions under the WTO, the threat of these sanctions provides incentives to allow forbearance (i.e. use the DSM). This causes countries to obtain outcomes that improve joint welfare. I also show that there are limits to forbearance that can be sustained by this mechanism - the fraction of simultaneous deviations against a single country that can be part of an equilibrium approaches a finite number as the number of countries participating in multilateral punishments becomes arbitrarily large. The results provide a theoretical basis for the DSM to offer prospective punishments rather than retroactive punishments and suggests a critical role for renegotiation.

JEL Classification: C73, D74, F10

Keywords: WTO, Tariffs, Political Economy of Trade Policy

*I wish to thank Kyle Bagwell, Dave Baron, Yossi Feinberg, Robert Feenstra, John Roberts and participants at the NBER Political Economy Public Finance summer institute and the Stanford Institute for Theoretical Economics for invaluable comments and suggestions. I am solely responsible for any errors or omissions.

†Graduate School of Business, Stanford University
“So should we succumb to a race to the bottom and a protectionism that history tells us that, in the end, protects no one?” -British Prime Minister, Gordon Brown, 2009

1 Introduction

Since the inception of the GATT in 1947 world trade has increased more than 14-fold, average tariffs have decreased in the US from a high of 35% in 1931 to 3.5% in 2007, and in Europe from roughly 35% in 1931 to 5.2% in 2007 over eight rounds of negotiations. There were 23 original signatories to the GATT and now the members of the WTO (its successor organization) stand at 153. By various measures one might say that the GATT and the WTO have been successful at promoting freer trade. But throughout the history of the GATT, doubts about its ability to withstand the forces of protectionism have periodically been raised.

The above quotation from the British Prime Minister was in response to the “Buy America” provisions President Barack Obama attempted to include in the package of spending aimed at stimulating the US economy in 2009. These provisions ultimately amounted to significant protection for US manufacturing companies, and the potential impact on international trade did not go unnoticed to US trading partners, who also raised trade barriers because of weaknesses in their economies. The Wall Street Journal commented in a March 2009 article that “A steady buildup of protectionist measures could ‘slowly strangle’ international trade.” The most recent global recession was not the first instance of the GATT agreement being questioned. In response to 1980’s global recession and subsequent protectionism, William Brock, the US Chief Trade Negotiator commented that, “The GATT system is in serious trouble.” This paper asks if there is a theoretical basis for these fears, or, more precisely, if there is a limit to cooperation that can be sustained in the WTO.

A natural place to begin answering this question is to have a theory of the WTO. In this this paper I present a theory based on three observations. I first observe that the success of the WTO has been in spite of its possessing very little enforcement

\[^2\text{Note that Rose (2004) has questioned the significance of the WTO in the increase in international trade, but other authors such as Subramanian and Wei (2007) have found evidence confirming that the WTO promotes trade. This paper proceeds by assuming the WTO plays a role in lowering trade barriers.}\]
power. Indeed the WTO has a very impressive dispute settlement mechanism that celebrated its 400th consultation in November 2009. But of the 400 cases, about half had been settled by the parties outside of the DSM, 186 had gone to litigation, and only about a dozen cases have actually resulted in a judgment being handed out. Some see the number of cases being brought to the DSM as a victory for the DSM, but the number appears to signal numerous departures from the GATT agreement with very little “punishing” taking place. In addition, countries found guilty often ignore GATT directives, as did the United States in the case of GATT-illegal antidumping duties imposed against Mexican cement in 1992. The strength of the WTO does not appear to be its ability to deter contravention of free trade.

The second observation is that while decision makers may have some private information, political economy shocks are largely public to all interested parties. For example, in 2002 President George W. Bush implemented steel “safeguards” under section 201 of the tariff act. The EU responded by drawing up a list of products on which its members would impose duties, and each product affected states that were most sensitive in President Bush’s upcoming re-election. These included, citrus fruits from Florida, apples and pears from Washington and Oregon, and steel from Pennsylvania, Ohio and West Virginia. In the words of the European trade commissioner, the tariffs were designed to “hit the US where it hurts”. Importantly, these tariffs were never implemented. The media is rife with examples of pressure groups vying for trade protection - catfish farmers in the Mississippi Delta area, cement manufacturers in southern states, and more recently US tire manufacturers clamoring for protection against Chinese imports.

The final key observation is that prior to the GATT, major trading countries had gone through periods of “global free trade”, followed by near complete breakdown in trading. For example when the UK repealed its Corn Laws in 1846 it set in motion the lowering of trade barriers across Europe (Conybeare (1987)). These barriers went up again with the onset of the first World War. After the first world war another attempt at lowering trade barriers was cut short by the 1930’s Smoot-Hawley Tariff Act. A similar pattern appears in earlier history with the Anglo-Hanse Trade Wars in the 1300-1700’s, and the Anglo French Trade Wars in 1664-1860 (Conybeare (1987)). In all these cases free trade ended amongst many trading countries, as trade restrictions were met with immediate retaliation from trading partners. This is in contrast to

---

2http:www.wto.org/englishnews_epres09_epr578_e.htm, WTO disputes reach 400 mark, November 6, 2009.
what we observe under the WTO. Under the WTO countries raise trade barriers (ie implement tariffs), but trading partners do not typically retaliate immediately. They take the case to the dispute settlement body and wait for the ruling. These rulings can take a number of years. In the meantime most countries reach a settlement and either reverse trade barriers or agree to some other trade concessions. 

I will present a model for which equilibria are consistent with these observations. I follow much of the literature by assuming major trading countries are engaged in a prisoners’ dilemma and governments occasionally experience political economy shocks. This follows work by Bagwell and Staiger (2002), Amador and Bagwell (2009), Rosendorff (2000) and others. I will assume these shocks are public information which is in contrast to Amador and Bagwell (2009) and Rosendorff (2000) who also examine the WTO as a self-enforcing institution. Given the WTO’s limited enforcement power we will assume that all trading countries are playing a Nash equilibrium of a repeated prisoners dilemma and that countries’ cooperation under the WTO agreement is simply a part of that equilibrium outcome. Consistent with the WTO’s limited power, in this sense it is not an actor in the game, but merely an equilibrium outcome. In this equilibrium, countries engaging in cooperation will allow partner countries to raise trade barriers in response to political economy shocks, and will withhold retaliation. The role of the WTO, and in particular the DSM, is therefore to allow forbearance. We feel this is a reasonable assertion as it reflects sentiments expressed by country leaders. In particular, the U.S. Trade negotiator, Robert Zoellick, in 2002 stated that “..rash use of sanctions could unleash a nuclear weapon on the [world] trading system.” (This was in response to the EU’s proposed list of retaliatory measures for the Bush steel tariffs.)

We will show that when countries discount the future enough, this form of cooperation is not possible with only two countries, but becomes possible with more than two countries. We will also show that the range of discount factors for which this cooperation is sustainable increases with the number of countries involved in the cooperation.

To illustrate briefly consider that the US and the EU are engaged in a cooperative trading relationship where it is understood that a breakdown in cooperation results in the two countries engaging in a trade war. These countries may be willing to

\[3\]The database put together by Horn and Mavroidis (2008) details these observations.

\[4\]This follows other work considering institutions-as-equilibria, such as Paul R. Milgrom and Weingast (1990), and Johns (2010).
continue the cooperation until one country, say the US, receives a political economy shock such that it is compelled to raise tariffs even given the threat of a future trade war. When politicians face term limits, it is easy to see how such a large shock may occur. If this shock is public, the EU can anticipate the US’s actions, and its best response will be to also raise tariffs. This would end the cooperation.

Now let us consider that it is not only the EU and the US in this cooperative agreement, but also Canada. Now when the US faces its political economy shock and is forced to raise tariffs against the EU, if the EU values its trading relationship with Canada, it may not retaliate immediately lest that trigger a downward spiral in all its trading relationships. Rather, if the shock is temporary, the EU may be willing to withhold retaliation during the period of the shock, and allow the US to return to normal trading relationships after the shock has passed. The EU will withhold retaliation today, knowing that should the EU face a similar shock in the future, the US will also withhold retaliation. So by Canada engaging in multilateral punishment with the US and the EU, it has allowed payoffs in the game between the US and the EU to move closer to a utilitarian optimum. The role of the WTO is simply to “signal” which countries are to engage in multilateral punishments, and coordinate on this kind of forbearance through the DSM. It is important to note that absent political economy shocks, adding countries will not have this effect, but will merely scale up incentive constraints preventing a self-enforcing multilateral agreement.

This approach of viewing the WTO as allowing countries breathing room in their trade obligations to reach mutually beneficial outcomes is complementary to Bagwell and Staiger (1990) who characterize subgame perfect equilibria in a model with two countries playing a tariff setting game. Each period a shock to trade flows is received, and in equilibrium countries raise tariff barriers in response to increases in imports. Such an equilibrium has not actually been observed with two countries acting independently, but in the WTO we do observe this equilibrium being played. Our results show that the multilateral punishment offered by the WTO creates additional incentives to allow forbearance in the face of shocks. The forces at work are similar to the multi-market contact forces first described by Bernheim and Whinston (1990). Unlike in Bernheim and Whinston (1990) the markets are identical, so in this paper incentive constraints are pooled across identical markets but in different “states”. This is what creates the slack in the incentive constraints.

This paper joins a growing literature on the analysis of the WTO as an institution. Most recently Chisik and Onder (2010) and Beshkar and Bond (2010) have made
important contributions to our understanding of the WTO’s institutional features. Other related papers include Ederington (2001), Maggi (1999), Hungerford (1991), Kovenoch and Thursby (1993), and Ossa (2009).

In section 2 I describe the benchmark model with only two countries and define the upper bound on the discount factor that prevents bilateral achievement of efficiency by withholding retaliation. In section 3 I extend this model to an arbitrary number of countries and characterize an equilibrium with cooperation. In section 6 I discuss the limits to cooperation, and section 5 concludes.

2 The Benchmark Model - Bilateral Trade relationships

Imagine two countries $i = 1, 2$ playing a tariff setting game over an infinite number of periods $t = 1, 2, \ldots$. Each period, players can choose an action, $a^i$, from the set $A = \{T, F\}$, where $T$ denotes high tariffs and $F$ denotes free trade. The stage game is a standard prisoner’s dilemma with payoffs given by the matrix below:

\[
\begin{array}{c|cc}
& T & F \\
\hline
T & 1,1 & \gamma^1, -5 \\
F & -5, \gamma^2 & 5,5 \\
\end{array}
\]

The value $\gamma^i \in \{6, 16\}$, can be thought of as political economy shocks. The probability that $\gamma^i = 16$ is $\varepsilon$. I assume $\varepsilon$ is less than $\frac{1}{2}$. Denote a profile of shocks as $\gamma = (\gamma^1, \gamma^2)$. Denote a profile of actions as $a = (a^1, a^2)$, and the payoffs for player $i$ is given by $\pi^i(a; \gamma)$. Denote a history of this game as $h$. The history will consist

---

5Notice that I could specify arbitrary payoffs of the form

\[
\begin{array}{c|cc}
& T & F \\
\hline
T & \alpha, \alpha & \gamma^i, \beta \\
F & \beta, \gamma^i & \eta, \eta \\
\end{array}
\]

with $\gamma^i \in \{\gamma, \overline{\gamma}\}$ and $\overline{\gamma} > \gamma$. General versions of the results hold as long as conditions for a prisoners’ dilemma game hold - $\gamma^i > \eta > \alpha > \beta$, and in addition $\overline{\gamma} - \alpha > \eta - \beta$. For ease of exposition I maintain the parameterization given above.

6It can be shown that allowing for correlated shocks does not change the results as long as the distribution of shocks among players remains symmetric.
of the history of past actions and shocks. Denote the space of histories as $H$, then a complete strategy for player $i = 1, 2$ is

$$\sigma : H \times \{6, 16\}^2 \to A^2.$$  

I will define a symmetric subgame perfect Nash equilibrium of this game as a strategy profile, $\sigma^* = (\sigma_1^*, \sigma_2^*)$, such that $\sigma_i^*$ maximizes the expected infinite horizon payoff for player $i$ in every subgame given $\sigma_j^*$.

Consider that we are interested in how close the outcomes of this game are relative to the utilitarian optimum which is simply the sum of the payoffs of the players. Define a strategy profile, $\bar{\sigma}$, that maximizes the sum of the payoffs for the game. It would have to satisfy for each stage game on the equilibrium path

$$\bar{\sigma}(16, 6) = (T, F), \text{ and } \bar{\sigma}(6, 16) = (F, T).$$

Let us say an equilibrium exhibits forbearance if it has this property. Although this condition is defined by payoffs, the definition of forbearance is about the action profile. By the folk theorem I know that a strategy profile with forbearance is supportable in equilibrium for high enough discount factors. Using the two player case as a benchmark, I wish to define an upper bound on the discount factor such that for all discount factors below the upper bound, forbearance cannot be part of the equilibrium. That is the action profile $(F, T)$ or $(T, F)$ is not played on the equilibrium path. Once this upper bound on discount factors is defined for the the two player case, I will show that as the number of players playing such bilateral prisoners' dilemma games increases, forbearance becomes possible in a subset of bilateral games.

**Lemma 1.** There exists a $\bar{\delta}(\epsilon) \in (0, 1)$ such that for all $\delta < \bar{\delta}(\epsilon)$, an equilibrium with forbearance does not exist. That is neither the action profile $(T, F)$ nor $(F, T)$ is played on the equilibrium path in any symmetric subgame perfect Nash equilibrium.

**Proof.** I must prove that there does not exist an equilibrium in which it is incentive compatible to play $(F, T)$ for player 1 or $(T, F)$ for player 2 for $\delta < \bar{\delta}(\epsilon)$. By symmetry I focus only on player 1’s incentives, and hence denote $\pi$ as player 1’s stage game payoff. Assume by way of contradiction that there does exist a $\delta < \bar{\delta}(\epsilon)$ and an
equilibrium strategy profile $\sigma^*$ such that $(F, T)$ is played along the equilibrium path. Define $\Pi(\sigma^*)$ as player 1’s expected continuation payoff for the game given strategy profile $\sigma^*$. Then for all subgames on the equilibrium path if $\sigma^*$ calls for $(F, T)$

$$(1 - \delta)\pi(F, T; \gamma) + \delta\Pi(\sigma^*(F, T)) \geq (1 - \delta)\pi(T, T; \gamma) + \delta\Pi(\sigma^*(T, T)).$$

Where $\sigma^*(T, T)$ are the continuation strategies dictated by the equilibrium after playing $(T, T)$.

I wish to compare the highest possible expected continuation payoff after playing $(F, T)$ to the lowest expected continuation payoff after playing $(T, T)$. This implies

$$(1 - \delta)(-5) + \delta[16\varepsilon + 6(1 - \varepsilon)] \geq 1$$

$$\Rightarrow \delta \geq \frac{6}{10\varepsilon + 11}.$$ 

So for $\delta < \frac{6}{10\varepsilon + 11}$ this is a contradiction.

Assumption 1. All discount factors satisfy

$$\delta < \frac{6}{10\varepsilon + 11} = \bar{\delta}(\varepsilon).$$

Assumption 1 states that the upper bound on the discount factor is $\bar{\delta}(\varepsilon) = \frac{6}{10\varepsilon + 11}$ which implies that with only two players $(F, T)$ or $(T, F)$ is not played on any equilibrium path, or in other words, forbearance is not possible. For all $\delta < \bar{\delta}(\varepsilon)$ players are not patient enough to sustain an equilibrium that allows players to obtain the payoff $(16, -5)$ on the equilibrium path. This is illustrated below in figure [1].

3 Multilateral Agreements

Now suppose that players’ discount factors satisfy $\delta < \bar{\delta}(\varepsilon)$ and at the beginning of the game players have the option of entering a self-enforcing agreement in which each country agrees to allow forbearance when their trading partner receives a high political economy shock. Two countries would be only too happy to sign such an
agreement if it were enforceable, but in the absence of an enforcement mechanism, this agreement would break down because withholding retaliation is not incentive compatible. I would like to know if this incentive is mitigated in the presence of a multi-country agreement. I begin with a three country illustration.

3.1 A Three country illustration

Now consider that there is a third country. Each of the three countries are now playing two identical prisoner’s dilemma games (one with each partner country) with payoffs given below.

<table>
<thead>
<tr>
<th></th>
<th>country 1</th>
<th>country 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T )</td>
<td>( F )</td>
</tr>
<tr>
<td>( T )</td>
<td>1,1</td>
<td>( \gamma_{1,2}^1 \cdot 5 )</td>
</tr>
<tr>
<td>( F )</td>
<td>-5,( \gamma_{1,2}^2 )</td>
<td>5,5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>country 1</th>
<th>country 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T )</td>
<td>( F )</td>
</tr>
<tr>
<td>( T )</td>
<td>1,1</td>
<td>( \gamma_{1,3}^1 \cdot 5 )</td>
</tr>
<tr>
<td>( F )</td>
<td>-5,( \gamma_{1,3}^2 )</td>
<td>5,5</td>
</tr>
</tbody>
</table>
Denote the action profile in the stage game between player $i$ and $j$ as $a_{i,j} = (a^i_{i,j}, a^j_{i,j})$, and let $a = (a_{1,2}, a_{1,3}, a_{2,3})$ denote a profile of stage game actions. As before let $\gamma$ denote a profile of shocks which is now $\gamma = (\gamma_{1,2}, \gamma_{1,3}, \gamma_{2,3})$, where $\gamma_{i,j} = (\gamma^i_{i,j}, \gamma^j_{i,j})$. Denote the entire space of shocks in one period by $\Gamma$. A strategy profile, $\sigma$, maps a history, $h$, and a profile of shocks into a pair of actions for each game. So

$$\sigma : H \times \Gamma \to A^2 \times A^2 \times A^2.$$  

The stage payoff to each country is given by the sum of payoffs in each of the games hence

$$\pi^i(a; \gamma) = \pi^i_{i,j}(a_{i,j}; \gamma^i_{i,j}) + \pi^i_{i,k}(a_{i,k}; \gamma^i_{i,k}).$$

Now I ask if an equilibrium with forbearance is possible. That is if $a_{i,j} = (T, F)$ or $a_{i,j} = (F, T)$ can be part of an equilibrium strategy profile for some $\delta < \delta(\varepsilon)$. Whereas this was not possible in an equilibrium with two players it is now possible in an equilibrium with 3 players.

**Proposition 1.** For $\varepsilon$ low enough, an equilibrium with forbearance exists in the game with three players. That is $(T, F)$ and $(F, T)$ are part of an equilibrium strategy profile for some $\delta < \delta(\varepsilon)$.

*Proof.* The proof is constructive. In what follows I drop the notation to show conditioning of the strategies on the history $h$ for brevity. Consider the following strategy profile, $\sigma^*$. Given no deviations in the history, $h$:

- $\sigma^*((6, 6), \gamma_{i,k}, \gamma_{j,k}) = ((F, F), a_{i,k}, a_{j,k})$, for any $\gamma_{i,k}, \gamma_{j,k}$.
- $\sigma^*((16, 16), \gamma_{i,k}, \gamma_{j,k}) = ((T, T), a_{i,k}, a_{j,k})$, for any $\gamma_{i,k}, \gamma_{j,k}$.
- $\sigma^*((6, 16), \gamma_{i,k}, \gamma_{j,k}) = ((F, T), a_{i,k}, a_{j,k})$ for all $\gamma_{i,k} \neq (6, 16)$, and any $\gamma_{j,k}$.
- $\sigma^*((6, 16), (6, 16), \gamma_{j,k}) = ((T, T), (T, T), a_{j,k})$ for any $\gamma_{j,k}$.

<table>
<thead>
<tr>
<th></th>
<th>$T$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>1,1</td>
<td>$\gamma^2_{2,3}$,-5</td>
</tr>
<tr>
<td>$F$</td>
<td>$-5, \gamma^3_{2,3}$</td>
<td>5,5</td>
</tr>
</tbody>
</table>
If there are any deviations in the history, \( h \), \( \sigma^*(\gamma) = ((T,T), (T,T), (T,T)) \) for all \( \gamma \).

The strategies essential dictate that countries maintain free trade as long as no country receives a high shock. If a country receives a high shock it can raise tariff barriers without its trading partner retaliating (allows forbearance) as long as its trading partner is not faced with two instances of high tariffs against it. If two of country \( i \)'s trading partners raise tariff barriers in response to shocks the equilibrium states that country \( i \) will raise tariffs in both its relationships, but return to cooperation after the shocks have passed. So on the equilibrium path there are episodes of all countries raising trade barriers in response to shocks by at least two countries.

The punishment in this equilibrium in the event of any deviation is to revert to high tariffs forever (or grim trigger). This reflects the idea that in the event the equilibrium being played in the WTO ceases to be played, countries would likely withdraw most concessions granted under the WTO (ie, revert to a global trade war). We are yet to observe this sort of collapse of the WTO, and the objective of this paper is show that the WTO can sustain incidences of high tariffs without this punishment being used.

It remains to show that this is an equilibrium by checking incentive constraints for each possible profile of shocks. By symmetry I will focus on the incentives of player 1, and I will start by calculating the expected payoff from the equilibrium strategies in a given period. Assuming no prior deviations in history, \( h \), the expected payoff for any player \( i \) is calculated in the appendix, and is

\[
\Pi(\sigma^*) = 10 + 2\varepsilon - 28\varepsilon^2 + 36\varepsilon^3 - 18\varepsilon^4. \tag{3}
\]

Now consider the expected payoff after deviating. The equilibrium strategies dictate playing \(((T,T), (T,T), (T,T))\) forever, giving players a payoff of 1 in each game, hence a total continuation payoff of 2. I wish to check the most binding incentive constraint. I will conjecture now and later prove that this is when the shock profile is either

\[
[(6,16), (16,16), \gamma_{2,3}] \text{ or } [(16,16), (6,16), \gamma_{2,3}],
\]
that is when player 1 is required to allow forbearance in one trading relationship, and is forced to retaliate in the other trading relationship. The equilibrium action profile is

\[ [(F, T), (T, T), a_{2,3}] \text{ or } [(T, T), (F, T), a_{2,3}]. \]

This gives player 1 a stage game payoff of \(-4\) (see figure 2 in the appendix). Consider player 1’s incentive to deviate to

\[ \tilde{\sigma} = [(T, T), (T, T), a_{2,3}] \]

This gives a stage game payoff of 2. Hence player 1 will have no incentive to deviate in this way as long as

\[
(1 - \delta)(-4) + \delta[10 + 2\varepsilon - 28\varepsilon^2 + 36\varepsilon^3 - 18\varepsilon^4] \geq 2
\]

\[
\Rightarrow \delta \geq \frac{3}{7 + \varepsilon - 14\varepsilon^2 + 18\varepsilon^3 - 9\varepsilon^4}.
\]

This lower bound on the discount factor is feasible if it is below \(\tilde{\delta}(\varepsilon)\), or if

\[
\frac{3}{7 + \varepsilon - 14\varepsilon^2 + 18\varepsilon^3 - 9\varepsilon^4} \leq \frac{6}{10\varepsilon + 11}
\]

\[
\Rightarrow \varepsilon \leq 0.2341.
\]

Now I must prove that this is the most binding constraint. Denote the equilibrium stage game payoff as \(\pi(\sigma^*)\), and the expected long run equilibrium payoff as \(\Pi(\sigma^*)\). Denoting an arbitrary deviation as \(\tilde{\sigma}\), the general condition for equilibrium strategies to be incentive compatible is

\[
(1 - \delta)\pi(\sigma^*) + \delta\Pi(\sigma^*) \geq (1 - \delta)\pi(\tilde{\sigma}) + \delta\Pi(\tilde{\sigma})
\]

\[
\Leftrightarrow \delta \geq \frac{\pi(\tilde{\sigma}) - \pi(\sigma^*)}{\pi(\tilde{\sigma}) - \pi(\sigma^*) + \Pi(\sigma^*) - \Pi(\tilde{\sigma})}.
\]
Notice that in the prescribed equilibrium for any deviation $\Pi(\tilde{\sigma}) = 2$ and $\Pi(\sigma^*) = 10 + 2\varepsilon - 28\varepsilon^2 + 36\varepsilon^3 - 18\varepsilon^4$. So I focus on the difference $\pi(\tilde{\sigma}) - \pi(\sigma^*)$ which is maximized at the most binding constraint. When the equilibrium strategy calls for playing $T$, there is no profitable deviation, so I focus on the equilibrium strategies that call for playing $F$. Consider first when $\sigma^* = (F,F)$. This only occurs when $\gamma_{i,j} = (6,6)$. A deviation would imply the strategy profile $\tilde{\sigma} = (T,F)$ hence

$$\pi(T,F) - \pi(F,F) = 6 - 5 = 1.$$ 

Now consider when $\sigma^* = (F,T)$. This only occurs when $\gamma_{i,j} = (6,16)$. A deviation would imply the strategy profile $\tilde{\sigma} = (T,T)$ hence

$$\pi(T,T) - \pi(F,T) = 1 - (-5) = 6.$$ 

These are the only possible deviations, so it’s clear that the binding constraint is when forbearance must be granted, or when $\sigma^* = (F,T)$, which implies a lower bound on the discount factor of

$$\delta(\varepsilon) = \frac{6}{6 + \Pi(\sigma^*) - \Pi(\tilde{\sigma})}. \quad (4)$$

This concludes the proof.

Now with three players there is a more severe punishment for not withholding retaliation than with two countries. Rather than having a single trading relationship break down, the threat is that multiple trading relationships break down, so it has the effect of *amplifying* the punishment.

Importantly, this amplification effect is absent if this game does not involve asymmetric shocks (and hence involve some variation in the strategies). Consider a simple game with no shocks, where $\pi(F,T) = 6$. Now there are no shocks, hence no reason to deviate from free trade. Consider now that $\delta(\varepsilon)$ is defined with two players to make sustaining free trade $(F,F)$ possible. Then I can show that adding more countries only scales up the incentive constraint and hence does not make the action profile $(F,F)$ feasible in any equilibrium. This is formalized in proposition 2.
**Proposition 2** (Amplification). Consider that $\varepsilon = 0$. If the action profile $(F,F)$ cannot be sustained in a game with two players, it cannot be part of an equilibrium for any number of players.

**Proof.** In this conjectured game the upper bound on the discount factor is defined so that $(F,F)$ is not supportable in equilibrium. Let us call the upper bound on the discount factor for this conjectured equilibrium $\tilde{\delta}$.

$$\tilde{\delta} = \frac{6 - 5}{6 - 5 + 5 - 1} = \frac{1}{5}$$

Now consider the $n$ player version where each country plays $(n - 1)$ games and payoffs are summed across games. For $(F,F)$ to be supportable it must be the case that

$$\delta \geq \frac{(n - 1)(6 - 5)}{(n - 1)(6 - 5 + 5 - 1)} = \frac{1}{5}.$$

This is a contradiction since I assumed $\delta < \tilde{\delta} = \frac{1}{5}$.

The intuition for the amplification result is that the shocks (and the implied strategies) generate some asymmetry in the payoffs, so when the constraint binds in a particular game, slackness in the other games can compensate. This intuition becomes even more stark if you consider the game with shocks but try to support only free trade. The binding constraint is then when the high shock occurs and players are to play $(F,F)$. Again the constraint is only scaled up with larger numbers of players so the amplification has no effect.

### 3.2 General multilateral agreements

It is not difficult to see that the strategies described for 3 players and the amplification effect generalizes to an arbitrary number of countries. I am interested to know the extent of forbearance that can be sustained as the number of countries in the agreement increases. Consider that there are $n = 3, 4 \ldots$ countries playing $G_n = \binom{n}{2}$ of these games. Each country plays $n - 1$ games and the size of the state space is $|\Gamma| = 4^{G_n}$. Now a strategy profile maps a history and a complete profile of shocks into a pair of actions for each game, hence
\[ \sigma : H \times \Gamma \to A^{2G_n}. \]

First we show that when the discount factor satisfies the upper bound such that two players cannot allow forbearance, we can find equilibria with forbearance in games with three or more players.

**Proposition 3.** For \( \varepsilon \) small enough, there exists a \( \delta < \delta(\varepsilon) \) such that an equilibrium in which forbearance is allowed in \( x \geq 1 \) trading relationships, exists.

**Proof.** The proof is again constructive. In this section we show that such an equilibrium exists, and in the next section, we consider the extent of forbearance for the class of equilibria considered. Consider a strategy profile analogous to the three country case where a country is allowed to retaliate if more than \( x \leq n - 1 \) trading partners raise tariffs against it because of high political economy shocks. To define this strategy profile I will need some extra notation.

Let \( t \) be the number of games in which player \( i \)'s trading partners receive a high political economy shock, and player \( i \) does not, so \( \gamma_{i,j} = (6, 16) \). Formally

\[
\sigma^* = \left( \begin{array}{c}
(F, F), a_{-(i,j)} \\
(T, T), a_{-(i,j)} \\
(F, T), a_{-(i,j)} & \text{if } t \leq x \\
(T, T), a_{-(i,j)} & \text{if } t > x
\end{array} \right)
\]

where \( I_{\gamma_{i,j} = (6,16)} \) is the indicator function which returns 1 if \( \gamma_{i,j} = (6, 16) \).

Let us also denote \( \gamma_{-(i,j)} \) as the shock profile excluding the shocks in the game with players \( i \) and \( j \), and let \( a_{-(i,j)} \) be the action profile excluding actions in the game with players \( i \) and \( j \). We again use the notation \( \sigma^* \) for the equilibrium to show that it is analogous to the three player equilibrium characterized above. (In fact, we can think of \( \sigma^*(n, x) \), and the equilibrium in section 3.1 is simply \( \sigma^*(3, 1) \).) If there are no prior deviations in history \( h \), the strategy profile is

- \( \sigma^*((6, 6), \gamma_{-(i,j)}) = ((F, F), a_{-(i,j)}) \),
- \( \sigma^*((16, 16), \gamma_{-(i,j)}) = ((T, T), a_{-(i,j)}) \),
- \( \sigma^*((6, 16), \gamma_{-(i,j)}) = ((F, T), a_{-(i,j)}) \) if \( t \leq x \),
- \( \sigma^*((6, 16), \gamma_{-(i,j)}) = ((T, T), a_{-(i,j)}) \) if \( t > x \).
If there are any deviations in history \( h, \sigma^*(\gamma) = ((T,T), \ldots, (T,T)) \) for all \( \gamma \).

The construction of the most binding constraint generalizes from the three player illustration. The lower bound on the discount factor for \( n \) countries is from equation (4)

\[
\delta(n, x) = \frac{6x}{6x + \Pi(\sigma^*; n, x) - (n - 1)}
\]  

Since each country plays \( n - 1 \) games, in the event of any deviation by a country the expected continuation deviation payoff is fixed at \( (n - 1) \).

Define the difference between the upper bound and the lower bound of the discount factor as

\[
\Delta(n, x) = \delta(\varepsilon) - \delta(n, x).
\]  

I am interested in values of \((n, x)\) such that \(\Delta(n, x)\) is positive which implies that these strategies constitute an equilibrium.

To conclude the proof I need to characterized the expected payoff and show that \(\Delta(n, x) > 0\) for some \(x\). The expected payoff is given by \(\Pi(\sigma^*; n, x)\) and is characterized in lemma 3.

\[
\Pi(\sigma^*; n, x) = (n - 1)[7\varepsilon - 11\varepsilon^2 + 5] + (-6)(n - 1)!(1 - \varepsilon + \varepsilon^2)^{n-1}\sum_{t=0}^{x} \frac{t}{t!(n - 1 - t)!} \left[ \frac{\varepsilon(1 - \varepsilon)}{1 - \varepsilon + \varepsilon^2} \right]^t.
\]  

Now let \(x = 1\). Then we have

\[
\Delta(n, 1) = \frac{6}{10\varepsilon + 11} - \frac{6}{6 + \Pi(\sigma^*; n, 1) - (n - 1)} > 0
\]

\[
\Rightarrow \Pi(\sigma^*; n, 1) > (n - 1) + 10\varepsilon + 5.
\]

Substituting for \(\Pi(\sigma^*; n, 1)\) we have
\[(n - 1)[7\varepsilon - 11\varepsilon^2 + 5] + (-6)(n - 1)\varepsilon(1 - \varepsilon)(1 - \varepsilon + \varepsilon^2)^{n-2} > (n - 1) + 10\varepsilon + 5.\]

\[\Rightarrow n - 1 > \frac{5 + 10\varepsilon}{4 + 7\varepsilon - 11\varepsilon^2 - 6\varepsilon(1 - \varepsilon^2)(1 - \varepsilon + \varepsilon^2)^{n-1}}.\]

This is true as long as \(\varepsilon\) is small enough. Taking the limit as \(\varepsilon \to 0\) I have \(n - 1 > 1.25\). Since I am considering \(n \geq 3\) for this section, this is proved true. This ends the proof of the proposition. \[\blacksquare\]

In the next section I explore the maximum value \(x\) can take as a function of \(n\), such that \(\sigma^*(n, x)\) is an equilibrium. That is, I examine the limits of forbearance in this class of equilibria.

## 4 Limits to Forbearance

The previous sections showed that by having more than two countries in an the agreement, the kind of forbearance induced by the WTO’s dispute settlement mechanism becomes possible, when it wasn’t possible with only two countries. In this section I examine the extent of forbearance possible in this class of equilibria as a function of the number of countries participating in the multilateral punishment.

Define the set \(X(n) = \{x : \Delta(n, x) > 0\}\). And define \(\bar{x}(n)\) to be the maximum value \(x\) can take with \(\Delta(n, x)\) positive. Formally,

\[\bar{x}(n) = \max\{x : x \in X(n)\}.\]

First, it is not difficult to show that \(\bar{x}(n) < n - 1\). This is the analog of the amplification result in proposition 2. To prove this I first need lemma 2, which says the expected payoff, \(\Pi(\sigma^*; n, n - 1)\), is exactly equal to \(\Pi(\sigma^*; 2, 1)(n - 1)\).

**Lemma 2.** \(\Pi(\sigma^*; n, n - 1) = \Pi(\sigma^*; 2, 1)(n - 1)\).

**Proof.** Given that \(x = n - 1\), payoffs for player \(i\) in the game with player \(j\) are not conditional on the payoffs in any other game. Hence the expected payoff to player \(i\) for the \(n - 1\) games in which player \(i\) plays is the sum of the expected payoff in each game. Each game which player \(i\) plays looks identical to the two player game with
only \( i \) and \( j \) when \( x = 1 \) for which the expected payoff is \( \Pi(\sigma^*; 2, 1) \). Summing this over \( n - 1 \) games gives the result. \footnote{This can also be proved by simplifying the expected payoff given in equation (12) in the Appendix.}

**Proposition 4.** If the equilibrium played is \( \sigma^* \), the maximum number of countries that can raise tariffs against a single country is strictly less than \( n - 1 \).

**Proof.** By way of contradiction, assume \( x = n - 1 \). Then the discount factor must satisfy

\[
\delta \geq \frac{6(n - 1)}{6(n - 1) + \Pi(\sigma^*; n, n - 1) - (n - 1)}.
\]

By lemma 2 I know

\[
\frac{6(n - 1)}{6(n - 1) + \Pi(\sigma^*; 2, 1)(n - 1) - (n - 1)} = \frac{6(n - 1)}{6(n - 1) + \Pi(\sigma^*; n, n - 1) - (n - 1)},
\]

so for this to be feasible in equilibrium it remains to show that

\[
\frac{6}{6 + \Pi(\sigma^*; 2, 1) - 1} < \delta.
\]

According to the equilibrium strategies, \( \sigma^* \), if \( n = 2 \) and \( x = 1 \), the expected payoff is given by

\[
\Pi(\sigma^*; 2, 1) = \varepsilon^2 + \varepsilon(1 - \varepsilon)(-5) + \varepsilon(1 - \varepsilon)(16) + (1 - \varepsilon)^2(5)
= 5 + \varepsilon - 5\varepsilon^2.
\]

Hence I have

\[
\frac{6}{10 + \varepsilon - 5\varepsilon^2} < \frac{6}{10\varepsilon + 11}
\]

\[
\iff 1 < -9\varepsilon - 5\varepsilon^2
\]

to obtain our contradiction. \footnote{This can also be proved by simplifying the expected payoff given in equation (12) in the Appendix.}
Next, I would like to know how $\pi(n)$ behaves as a function of $n$. Proposition 5 gives us that $\pi(n)$ is an increasing function of $n$ as we would expect.

**Proposition 5.** The maximum number of simultaneous shocks that country $i$ can sustain in equilibrium, $\pi(n)$, is an increasing function of the number of countries, $n$.

**Proof.** To characterize $\pi(n)$ I need to find the maximum $x$ such that $\Delta(n;x) > 0$. This implies

$$(n - 1)(7\epsilon - 11\epsilon^2 + 4) > (6)(n - 1)!(1 - \epsilon + \epsilon^2)^{n-1} \sum_{t=0}^{x} \frac{t}{t!(n-1-t)!} \left[ \frac{\epsilon(1-\epsilon)}{1-\epsilon+\epsilon^2} \right]^t + x(10\epsilon + 5).$$

The first term on the right hand side is independent of $x$ and becomes very small relative to the last term as $x$ increases, hence I can ignore this term and show that the upper bound on $x$ is approximately

$$\pi(n) \approx (n - 1) \left[ \frac{7\epsilon - 11\epsilon^2 + 4}{10\epsilon + 5} \right].$$

This is clearly an increasing function of $n$.

Finally, a key proposition in the paper is that there is a limit to forbearance that this equilibrium can support in the sense that the fraction of countries which can withstand simultaneous tariff barriers being raised against them approaches a finite number. Proposition 6 gives this result.

**Proposition 6** (Limits to forbearance). The ratio $\frac{\pi(n)}{n}$ is an increasing function of $n$ and approaches a finite number as $n$ gets large. In this sense there is an upper bound to the level of forbearance that can be achieved via multilateral punishments.

**Proof.** I know that $\lim_{n \to \infty} \frac{n-1}{n} = 1$. So from equation 10

$$\lim_{n \to \infty} \frac{\pi}{n} = \frac{7\epsilon - 11\epsilon^2 + 4}{10\epsilon + 5}.$$  

This limit is a decreasing function of $\epsilon$. For $0 \leq \epsilon \leq 0.5$, the limit is between 0.8 and 0.475. So, intuitively, with a smaller fraction of shocks occurring, more
cooperation can be sustained. With the specified payoffs, at most, we would expect any single country to still withhold retaliation when 80% of its trading partners raise tariffs against them, and at worst 45%. These numbers appear to be encouraging as they represent a significant amount of trading relationships.

5 Conclusion

This paper describes the mechanism by which the WTO sustains cooperation among its member countries. The important feature is the forbearance, or withholding of retaliation, which is operationalized through the WTO’s Dispute Settlement Mechanism. I show that an equilibrium with this forbearance becomes more likely as the number of countries in the agreement grows, but as the number of countries grows the fraction of trading partners that can raise tariffs in a period against a single country approaches a finite number.

Interestingly, any punishment in this equilibrium is not retroactive, but is prospective (that is, punishments do not calculate damages going backward, and pay a lump sum, but allow withholding of tariff concessions going forward). If these punishments were retroactive they would not allow countries to take advantage of higher payoffs when high shocks occur (since the future cost of this behavior would be prohibitive, and the time-consuming process of going to the dispute settlement mechanism that generates the cooperation would be nullified). The prospective nature of the punishment, and the act of withholding most punishment allows countries to obtain payoffs that improve joint utility, and lifetime expected payoff.

Necessarily on the equilibrium path there are shock profiles for which countries do retaliate temporarily when enough trading partners receive high shocks against them. This suggests that in periods of multiple shocks, for example during a global recession, countries should be excused from the obligations under the WTO across several relationships if necessary. This is where the ability to renegotiate commitments becomes important in the WTO. The flexibility afforded by renegotiation allowed by GATT Article XXVII allows the WTO to adapt to episodes of shocks that may not have been conceived at its inception. In the constructed equilibrium countries with a high enough number of shocks against them retaliate in all of their trading relationships. There exist equilibria with potentially higher expected payoffs in which countries may retaliate in a limited number of relationships. These kinds of
equilibria were not discussed in this paper given that they seems less likely to be operationalized in a real world institution because if they are to be symmetric it would involve mixing over relationships in which retaliation occurs, and if it is asymmetric, it distorts incentives for countries that become more frequently retaliated against.

Finally, the results in this paper hinge on assuming that trading partners are close to symmetric and are in fact playing a prisoner’s dilemma game. Symmetry, in the context of the model, relates to magnitude of payoffs, discount factors, and political economy shocks. This would apply to the original signatories to the GATT - large, mostly democratic countries - but importantly may not apply to some newer members, notably China, an increasingly important member of the WTO.

China appears to satisfy the condition for symmetry in magnitude of payoffs, but not necessarily for discount factors and political economy shocks since China is not a democracy. This seems to be supported by recent comments by Zhang Guobao, the vice chairman of the Chinese National Development and Reform Commission, in reaction to the US proposed investigation into China subsidizing clean technology - "I have been thinking: What do the Americans want? [...] Do they want fair trade? Or an earnest dialogue? Or transparent information? I dont think they want any of this. I think more likely, the Americans just want votes."[8] The comment reflects an absence of symmetric political economy shocks in China. However, this is not to say that China cannot play strategies that mimic forbearance, as the Chinese government will face economic shocks that make raising tariff barriers more efficient that maintaining free trade. These shocks, however, must be public and verifiable (as the shocks in this model are, crucially, common knowledge). Because of China’s growing importance, this model suggests that a forbearance strategy by China could significantly contribute to maintaining the equilibrium being played in the WTO.

Other notable exceptions to the symmetry assumption are smaller countries who are now currently a part of the WTO. Absent political economy shocks unilateral free trade is optimal for countries that are small enough, so the prisoners’ dilemma is not the appropriate game to model their incentives for being a part of the WTO. Indeed for larger countries the threat of losing these trading relationship seems that it would provide greater incentive to uphold WTO commitments, but less so than the major trading partners. One could consider smaller countries as having different discount factors or smaller payoffs, and then some natural questions to ask are how

---

are the limits to the equilibrium cooperation affected when discount factors are not identical, or payoffs are not symmetric. This I leave for future work.
6 Appendix

6.1 Expected payoff calculation for three player illustration

Notice with three players there are $4^3 = 64$ possible states. Calculation of the expected payoff involves calculating the payoffs in each state given the equilibrium strategies. I do this with help of spreadsheet and simplified with the help of Maple mathematical software.

By symmetry I focus on the payoffs of player 1. First payoffs are calculated for each state given the equilibrium strategies. These are given by

\[
\begin{array}{cccc}
\text{Player 2 v 3} & (6,6) & \text{Player 1 v 3} & \text{Player 2 v 3} \\
\text{shocks} & 6,6 & 6,16 & 6,16 & 6,16 \\
\text{payoff} & 6,6 & 6,16 & 6,16 & 6,16 \\
5 & 10 & 21 & 0 & 6 \\
16 & 16 & 32 & 11 & 17 \\
6 & -5 & 8 & 4 & 2 \\
16 & 16 & 4 & 2 & 2 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Player 2 v 3} & (16,6) & \text{Player 1 v 3} & \text{Player 2 v 3} \\
\text{shocks} & 6,6 & 6,16 & 6,16 & 6,16 \\
\text{payoff} & 6,6 & 6,16 & 6,16 & 6,16 \\
5 & 10 & 21 & 0 & 6 \\
16 & 16 & 32 & 11 & 17 \\
6 & -5 & 8 & 4 & 2 \\
16 & 16 & 4 & 2 & 2 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Player 2 v 3} & (6,16) & \text{Player 1 v 3} & \text{Player 2 v 3} \\
\text{shocks} & 6,6 & 6,16 & 6,16 & 6,16 \\
\text{payoff} & 6,6 & 6,16 & 6,16 & 6,16 \\
5 & 10 & 21 & 0 & 6 \\
16 & 16 & 32 & 11 & 17 \\
6 & -5 & 8 & 4 & 2 \\
16 & 16 & 4 & 2 & 2 \\
\end{array}
\]

Notice the shaded payoffs correspond to $\sigma^*((6,16),(6,16),\gamma_{j,k}) = ((T,T),(T,T),a_{j,k})$.

Now I calculate corresponding probabilities where $\varepsilon$ is substituted for $e$. These are given by
Let $\gamma_i,j$ be the number of games in which player $i$ receives a high political economy shock but player $i$’s partner does not, so $\gamma_i,j = (16,6)$ and let $v$ be the number of games in which both player $i$ and player $i$’s trading partner receives a high political economy shock, so $\gamma_i,j = (16,16)$. Formally

$$\Pi(\sigma^*; n, x) = (n-1)[7\varepsilon - 11\varepsilon^2 + 5]$$

$$+ (-6)(n-1)!\varepsilon(1 - \varepsilon + \varepsilon^2)^{n-1} \sum_{t=0}^{x} \frac{t}{t!(n-1-t)!} \left[ \varepsilon(1 - \varepsilon) \right]^t.$$

**Proof.** Let $u$ be the number of games in which player $i$ receives a high political economy shock but player $i$’s partner does not, so $\gamma_i,j = (16,6)$ and let $v$ be the number of games in which both player $i$ and player $i$’s trading partner receives a high political economy shock, so $\gamma_i,j = (16,16)$. Formally
\[ u = \sum_{j \neq i} I_{\gamma_{i,j}=(16,6)} \]

\[ v = \sum_{j \neq i} I_{\gamma_{i,j}=(16,16)}. \]

Since all players are symmetric, I can consider the shocks in games in which player \( i \) plays as summarized by \((t, u, v)\). Hence the unconditional probability of \((t, u, v)\) is given by

\[ Pr[t, u, v] = [\varepsilon(1 - \varepsilon)]^t[\varepsilon(1 - \varepsilon)]^u[\varepsilon^2]^v[(1 - \varepsilon)^2]^{n-1-t-u-v} \]

\[ \left( \begin{array}{c} n-1 \\ t \end{array} \right) \left( \begin{array}{c} n-1-t \\ u \end{array} \right) \left( \begin{array}{c} n-1-t-u \\ v \end{array} \right). \]

When \( \gamma_{i,j} = (16, 6) \) the payoff is conditional on whether more than \( x \) of player \( j \)'s trading partners receive a shock against \( j \). So I also need to write down these conditional probabilities. Assume \( u \geq 1 \) and let us write down the probability of \((t, u, v)\) conditional on \( w_j \geq 1 \) of player \( j \)'s trading partners having a high shock against \( j \), so \( \gamma_{k,j} = (16, 6) \). Define \( w_j \) formally as

\[ w_j = \sum_{k \neq j} I_{\gamma_{k,j}=(16,6)} \]

\[ Pr[t, u, v; w_j = k] = [\varepsilon(1 - \varepsilon)]^t[\varepsilon(1 - \varepsilon)]^u[\varepsilon^2]^v[(1 - \varepsilon)^2]^{n-1-t-u-v} \]

\[ \left( \begin{array}{c} n-1 \\ t \end{array} \right) \left( \begin{array}{c} n-1-t \\ u \end{array} \right) \left( \begin{array}{c} n-1-t-u \\ v \end{array} \right) \]

\[ [\varepsilon(1 - \varepsilon)]^k[1 - \varepsilon(1 - \varepsilon)]^{n-2-k} \left( \begin{array}{c} n-2 \\ k \end{array} \right). \]

Now let us define the probability of \((t, u, v)\) conditional on player \( j \) receiving a high shock in \( x \) or less of his trading relationships.
Define the probability of \((t, u, v)\) conditional on trading partner \(j\) and trading partner \(k\) receiving a high shock in \(x\) or less of their trading relationships.

\[
Pr[t, u, v; w_j \leq x] = \varepsilon(1 - \varepsilon)^t \varepsilon(1 - \varepsilon)^u \varepsilon(1 - \varepsilon)^v (1 - \varepsilon)^{(n-1) - t - u - v}
\]

\[
\left( \begin{array}{c} n-1 \nonumber \\
\end{array} \right) \left( \begin{array}{c} n-1-t \nonumber \\
\end{array} \right) \left( \begin{array}{c} n-1-t-u \nonumber \\
\end{array} \right)
\]

\[
\sum_{k=0}^{x-1} \varepsilon(1 - \varepsilon)^k [1 - \varepsilon(1 - \varepsilon)]^{n-2-k} \left( \begin{array}{c} n-2 \nonumber \\
\end{array} \right).
\]

Now let us define the probability that, of the \(u\) games in which player \(i\) gets a high shock against his trading partner, in \(w \leq u\) of these games player \(i\)’s trading partner does not have more than \(x\) high shocks against them. So

\[
w = \sum_{j \neq i} I(\gamma_{i,j} = (16, 6) \text{ and } w_j \leq x).
\]
\[
Pr[t, u, v; w] = [\varepsilon(1 - \varepsilon)]^t [\varepsilon(1 - \varepsilon)]^u [\varepsilon^2]^v [(1 - \varepsilon)^2]^{n-1-t-u-v}
\]
\[
\binom{n-1}{t} \binom{n-1-t}{u} \binom{n-1-t-u}{v} \binom{u}{w} \sum_{r_1=0}^{x-1} \left\{ [\varepsilon(1 - \varepsilon)]^{r_1} [1 - \varepsilon(1 - \varepsilon)]^{n-2-r_1} \binom{n-2}{r_1} \right\}
\]
\[
\sum_{r_2=0}^{x-1} \left\{ [\varepsilon(1 - \varepsilon)]^{r_2} [1 - \varepsilon(1 - \varepsilon)]^{n-2-r_2} \binom{n-3}{r_2} \right\} \ldots
\]
\[
\sum_{r_{w-1}=0}^{x-1} \left\{ [\varepsilon(1 - \varepsilon)]^{r_{w-1}} [1 - \varepsilon(1 - \varepsilon)]^{n-2-r_{w-1}} \binom{n-w}{r_{w-1}} \right\}
\]
\[
\sum_{r_w=0}^{x-1} \left\{ [\varepsilon(1 - \varepsilon)]^{r_w} [1 - \varepsilon(1 - \varepsilon)]^{n-2-r_w} \binom{n-1-w}{r_w} \right\}
\]

This simplifies to

\[
Pr[t, u, v; w] = \varepsilon^{t+u+2v}(1 - \varepsilon)^{2(n-1-v)-t-u}(1 + \varepsilon - \varepsilon^2)^{\sum_{r=0}^{w-1} n-2-r} (n-1)! \over t!v!w!(n-1-t-u-v)!u-w)!.
\]

According to the equilibrium strategies for \( t \leq x \), \( \sigma^*((6, 16), \gamma_{-(i,j)}) = ((F, T), a_{-(i,j)}) \), and if \( t > x \), \( \sigma^*((6, 16), \gamma_{-(i,j)}) = ((T, T), a_{-(i,j)}) \). So the expected payoff is

\[
\Pi(\sigma^*; n, x) = \sum_{t=0}^{x} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-u} \sum_{w=0}^{u} \{ Pr[t, u, v; w].
\]
\[
\left\{ -5t + 16w + (u-w) + v + 5(n-1-t-u-v) \right\} + 
\]
\[
\sum_{t=x+1}^{n-1} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-u} \sum_{w=0}^{u} \{ Pr[t, u, v; w].
\]
\[
\left\{ t + 16w + (u-w) + v + 5(n-1-t-u-v) \right\}.
\]

This simplifies to
\[ \Pi(\sigma^*; n, x) = \sum_{t=0}^{x} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-t-u} \sum_{w=0}^{u} \{ Pr[t, u, v; w] \} \cdot \left\{ -10t + 15w - 4u - 4v + 5(n-1) \right\} + \]
\[ \sum_{t=x+1}^{n-1} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-t-u} \sum_{w=0}^{u} \{ Pr[t, u, v; w] \} \cdot \left\{ -4t + 15w - 4u - 4v + 5(n-1) \right\} . \]

Given that the probabilities of other trading partners having more than \( x \) high shocks against them is vanishingly small, in what follows I ignore the conditioning on \( w \) hence

\[ \Pi(\sigma^*; n, x) = \sum_{t=0}^{x} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-t-u} \{ Pr[t, u, v] \} \cdot \left\{ -10t + 11u - 4v + 5(n-1) \right\} + \]
\[ \sum_{t=x+1}^{n-1} \sum_{u=0}^{n-1-t} \sum_{v=0}^{n-1-t-u} \{ Pr[t, u, v] \} \cdot \left\{ -4t + 11u - 4v + 5(n-1) \right\} . \]

This simplifies to

\[ \Pi(\sigma^*; n, x) = (n-1)[7\varepsilon - 11\varepsilon^2 + 5] + (-6)(n-1)!(1-\varepsilon + \varepsilon^2)^{n-1} \sum_{t=0}^{x} \frac{t}{t!(n-1-t)!} \left[ \frac{\varepsilon(1-\varepsilon)}{1-\varepsilon + \varepsilon^2} \right]^t, \]

and further simplifies to

\[ \Pi(\sigma^*; n, x) = (n-1)[7\varepsilon - 11\varepsilon^2 + 5] + (-6)(n-1)!(1-\varepsilon + \varepsilon^2)^{n-1} \sum_{t=0}^{x} \frac{t}{t!(n-1-t)!} \left[ \frac{\varepsilon(1-\varepsilon)}{1-\varepsilon + \varepsilon^2} \right]^t. \]
The intuition for the expected payoff is as follows. Except for games in which a shock of \((6, 16)\) occurs for player \(i\), the payoffs in each game is (approximately) independent of payoffs in other games. Hence the expected payoffs to games when the shock is \(\gamma_{i,j} \in \{(6, 6), (16, 6), (16, 16)\}\) can be scaled up. This is the first term in the expected payoff. The last term represents the decrement to the expected payoff from increasing \(x\). By this it is clear that this expected payoff is a decreasing function of \(x\). In the proof I ignore conditioning the payoff when \(\gamma_{i,j} = (16, 6)\) for simplicity. This is why it is approximately.
References


