Distribution of power within the IMF: when does preference mean voice?*

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Abstract

Using the preference-based approach to power analysis of the International Monetary Fund from Aleskerov, Kalyagin & Pogorelskiy (2008), which allows for estimating the power of the IMF members within the Executive Board and the Fund in general through the existing constituency system, we explore a new model of members' preferences to coalesce. The preferences in this model are based on the data on countries' bilateral trade.

The present results of voting power analysis (as of May 2009) are compared with those produced by the classical power indices by Banzhaf and Penrose. We show that the greater the majority voting rule, the more the preferences to coalesce matter for countries with a small number of votes.

1 Introduction

Formal analysis of voting power distribution in the functioning voting bodies is based on the likelihood of a situation in which a voter is *decisive*, i.e., is able to swing a division of the voting body into those voting 'yes' and 'no' in the voter's preferred direction. One could argue that this is when the actual power is revealed.

On the other hand, a member's voting power is still commonly viewed as a share of the total number of votes she owns. Although intuitively compelling, the latter statement is inconsistent with the aforementioned idea of power since having a non-zero voting share is not sufficient for being able (at least formally) to affect the collective decision. Many artificial and real cases demonstrate this discrepancy (see Leech (2002), Aleskerov, Kalyagin & Pogorelskiy (2008)), but its inappropriateness can also be inferred from the following trivial example.

Suppose we have a voting body where not all voters have the same voting shares and the decision making rule is simple majority, i.e., at least half[2] of the total votes is needed to pass a decision. Select any voter and assume that her power can be measured as her voting share.

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[2] Strictly speaking, this number must be 50% +1 vote.

Consider a situation in which the voting body remains the same, but the decision making rule has changed: now unanimity of votes is required to pass a decision. Clearly, in the latter case everybody has the same power to affect the collective decision: all votes are needed. The voting shares, however, have not changed. This example shows that in addition to voting shares, power also depends on the quota[3].

To capture all factors defining power it is generally accepted to use a special measure called *power index*. The most widely known classical power indices are those introduced by Penrose (1946), Banzhaf (1965), Coleman (1971) and Shapley-Shubik (1954). It should be noted that these indices do not use any additional information about the voting body, just vote distribution and the quota, and thus are often referred to as *a priori* power indices. For a power analysis of functioning voting bodies to have this basic information is not enough: one also needs to know the probability distribution on possible divisions of the body.

A fairly reasonable, but not indisputable way to derive this distribution is by introducing voters' preferences to coalesce into the model. The major assumption of the approach is that these preferences determine voters' willingness to vote together and hence to form coalitions. There are some recent findings supporting this assumption at least in laboratory experiments (see Aleskerov, Belianin & Pogorelskiy (2009)).

This paper applies the preference-based voting power analysis to the International Monetary Fund (IMF). The Fund itself is an interesting object for research not only because of its significant role in the world economy, but also because of the representativeness issues that have been accompanying it since its creation in 1944 (for more details, see Mikesell (1994)). Several papers have been published that concentrated on *a priori* voting power analysis of the current governing bodies of the IMF (see Leech (2002), Alonso-Meijide & Bowles (2005)), while another strand of literature used the same approach (i.e., classical power indices) for analysis of the changes in power distribution arising from some modifications in the Fund's structure and the rules of governance (a recent work is Leech & Leech (2009)). Concerning the preference-based power indices, the only paper to employ this methodology for the power analysis of the IMF was Aleskerov, Kalyagin & Pogorelskiy (2008).

In this paper we use a new model of the countries' preferences to coalesce. As defining factor in this model we consider the countries' bilateral trade.

The paper is organised as follows. Section 2 describes the IMF governance structures, in Section 3 the main notions of the model are given, Section 4 deals with the voting power

[3] Note that *quota* here means a number of votes required for a decision to be taken; it is not related to a country's quota (the term adopted by the IMF), which has several functions: in particular, it defines a country's number of votes. The difference between these two notions can be easily seen from the context, however, we will emphasise it, where appropriate.

analysis in the Fund and its results, Section 5 concludes. Appendix A contains the power indices for the IMF members for both models.

2 The Governance of the Fund

This paper is primarily concerned with a single aspect of the IMF governance – the one related to members' voting power they possess in the process of decision making. This view is inevitably incomplete with regards to other administrative and economical issues that have been shaping the current organization of the IMF managerial structures. A detailed description of the governance process of the Fund can be found in van Houtven (2002) and shall not be reproduced here. Note also that the IMF is currently being reformed and more recent information is regularly published on the official website of the IMF: www.imf.org. With regard to this, all the factual information in the paper is as of May 2009.

The main voting body of the IMF is the Board of Governors comprising one Governor and one Alternate from each member country. The decisions in the Board are taken using the simple majority rule for most cases except some special ones where the qualified majority of 70% and 85% is used. Every Governor casts all the votes allotted to her state as a unit.

The administration and maintenance of the Fund are vested in the Managing Director and the Executive Board, of which 24 Executive Directors (EDs) and Alternate Executive Directors, (AEDs) are the main staff. It should be noted that "By-Laws of the IMF" and "Rules and Regulations of the IMF" allow the Chairman to "ordinarily ascertain the sense of the meeting in lieu of a formal vote" and this method is widely used by the Executive Board; therefore, officially almost all decisions are reached by consensus. However, in practical terms this means accepting the decision when the informal agreement of the majority required is met (van Houtven (2002)). This allows us to assume that voting, either explicit or not, takes place in any ballot.

The Board of Governors may delegate to the Executive Board authority to exercise any powers, except those conferred directly by the Articles of Agreement[4] on the Board of Governors. In fact, the Board of Governors has made the maximum possible delegation of authority to the Executive Board (van Houtven (2002)). Nevertheless, several categories of decisions, most of which relate to adjustment of quotas, allocation/cancellation of SDRs[5], the size of the Board, and so on, cannot be delegated to the Executive Board. Nearly all of such decisions require the support of the majority of 85% of the total votes. These few cases of

[4] The full text of the Articles of Agreement is available at www.imf.org/external/pubs/ft/aa/index.htm.
[5] SDR stands for Special Drawing Right: a monetary unit of the IMF, which value is defined by a basket of currencies.

decision making in the Board of Governors are left for future research. At the same time, the results of *a priori* voting power analysis by Leech (2002) showed little difference between the power distributions within the two governing Boards. Our analysis therefore was limited to the Executive Board as the primary governing body of the IMF.

Of the total of 24 EDs in the Executive Board, five are appointed by the countries with the largest quotas (the USA, Japan, Germany, France and the UK), while the remaining nineteen are elected by the members of the Board of Governors who do not represent the abovementioned five states in the Fund. There are several rules of elections of the EDs in the Articles of Agreement that make most countries having insufficient shares of the total votes form *constituencies* to be represented by the 19 elected EDs. These constituencies are not fixed structures; at the biannual elections of Executive Directors any country can change a constituency by simply casting its votes for another ED. Woods & Lombardi (2006) present several examples of such precedents.

Each appointed ED can cast as many votes as was allotted to her state. Each elective ED can cast as many votes as was received by her during elections. All votes an ED has can only be cast as a unit, and this is one of the sources of issues with representativeness.

3 The model

Below we briefly repeat some notation from Aleskerov, Kalyagin & Pogorelskiy (2008) in order to make this paper self-contained.

3.1 Power indices taking into account members' preferences

Let V be a generic constituency comprising l countries that uses a decision making rule with a quota[6] set at q. We assume that no country is allowed to abstain from voting. A *coalition* here is conventionally understood as a subset of countries voting in the same way. $\omega \subseteq V$ is a *winning* coalition, if it can determine the collective decision of V without other countries' votes, i.e., the sum of the votes of countries from ω is not less than q. Otherwise, the respective coalition is a *losing* one.

A country in a winning coalition is called *pivotal* if the coalition ceases to be winning in case of this country's exit. A coalition ω is a *swing* for country $i \notin \omega$, if ω is a losing one while $\omega+\{i\}$ is a winning one.

[6] Note that quota here means the decision making rule and not a country's IMF quota. For constituencies we assume that the simple majority rule is used for taking decisions, i.e., q is determined by $\left\lceil \frac{v_T}{2} \right\rceil$ for an odd v_T or $\frac{v_T}{2}+1$, for an even v_T, where v_T is the total number of votes.

Following Felsenthal & Machover (2002), to evaluate the power of every country in the Executive Board we adopt the approach described below:

- Consider a system of indirect voting.

- The total power of each country is defined by its power in a given constituency and the power of the constituency in the Executive Board. Namely, the power index $\pi(i,V)$ representing *i*'s total power in the Board (with *i* belonging to the constituency *V*) is the arithmetic product of the country's power index in the given constituency (denoted by $\alpha(i)$) and the power index of constituency V in the Executive Board (denoted by $\kappa(V)$) .

At the constituency level we use a modification of the Penrose power index that allows for taking into account the agents' preferences to coalesce.

Assume that the cardinal preferences of each country to coalesce with any other country of the same constituency are given in the respective preference profile $\vec{P}_i = (p_{i1}, \dots, p_{ii-1}, p_{ii+1}, \dots, p_{in})$, where n is the total number of members of the Fund, and $p_{ij} \in [0,1], i,j \in \{1,..,n\}$. Define the mean intensity of country *i*'s connections with other members of the same coalition ω as follows (Aleskerov (2006)).

$$f_i^+(\omega) = \frac{\sum_{j\in\omega} p_{ij}}{|\omega|} \qquad (1)$$

Let us define the mean intensity of connections of other countries of ω with *i* by

$$f_i^-(\omega) = \frac{\sum_{j\in\omega} p_{ji}}{|\omega|} \qquad (2)$$

and the mean intensity of connections within a coalition by

$$f(\omega) = \frac{\sum_{i\in\omega} \frac{1}{2}(f_i^+(\omega\backslash\{i\}) + f_i^-(\omega\backslash\{i\})}{|\omega|} \qquad (3)$$

The intensity $f_i^+(\omega)$ may be interpreted as a likelihood of country *i* joining ω (if this coalition has formed) based on *i*'s pairwise propensities (defined in accordance with the preference profile of the country *i*) to join each country of the coalition.

The intensity$f_i^-(\omega)$ is similar to (1) with the distinction that in (2) the preferences of other countries about *i* joining their coalition are taken into account. In some sense this may be understood as acceptance of the country *i* by the members of coalition ω.

The intensity $f(\omega)$ characterizes the average intensity of connection among the countries of coalition ω. It may be seen as likelihood that coalition ω acts as a bloc, i.e., all the members of the coalition vote in the same way. Note that when calculating the sum in the numerator of (3), for each agent *i* evaluated is her mean intensity of connections with other members of the coalition, i.e. $\omega\backslash\{i\}$.

The constituency level power index α of a country *i* is then defined as

$$\alpha(i) = \begin{cases} \sum_{\substack{\omega \subseteq V: \\ \omega \text{ is a swing } for\ i}} \frac{f_i^+(\omega)}{2^{l-1}} & if\ v(i) < q \\ 1 & otherwise \end{cases} \qquad (4)$$

where *l* is the number of countries of the constituency, *v(i)* is the number of votes of country *i*, q is the quota for a decision to be taken. This is an absolute power index. Note that if we assume for all coalitions ω $f_i^+(\omega) = 1$, (4) becomes the Penrose (1946) power index. Further, for every country the Banzhaf (1965) power index can be derived from the Penrose one by normalizing the respective value by the total sum of the Penrose indices for all countries.

The index (4) can be interpreted as a ratio of the summarised intensity of *i*'s potentially possible connections with those coalitions which are swings for *i*, to a maximum possible value for the intensity of connections in a given constituency providing that *i* is not a dictator. If, on the other hand, the number of *i*'s votes exceeds the quota, her preferences over coalescing with other participants lose their meaning because *i* can determine the decision by herself. Hence, provided that the quota is greater than 50% of the total votes, the voting power of other members is always zero (they cannot swing any vote), and *i* is decisive (i.e., completely define the outcome of the ballot) with certainty.

At the level of the Executive Board we define the power of a constituency in probabilistic terms. We are going to assume that the constituencies vote independently, while their probabilities to vote for or against a decision depend on their members' preferences to coalesce in the following way: the probability for a constituency *V* to vote "yes" is given by

$$\Pr(V\ votes\ 'yes') = \frac{\sum_{\omega\ is\ winning} f(\omega)}{2^l} \qquad (5)$$

In other words, for every winning coalition ω the function *f (ω)* defines a degree of inner consolidation reflected in the probability of forming such a coalition. Note that if for all coalitions $f(\omega) = 1$, then formula (5) gives "The Power of the Body to Act" (PTA) – a power index devised by Coleman (1971).

Our assumption that different constituencies vote independently results in ruling out the possibility of constituencies to form alliances. The reason is the need to have aggregate preferences to coalesce over constituencies. Such aggregation seems inadequate for the preference factors considered due to diversity of countries in existing constituencies.

The power of a constituency V at the level of the Executive Board is then defined as its probability of being decisive, given by

$$\kappa(V) = \sum_{\substack{S \subseteq N: \\ S \text{ is a swing for } V}} \prod_{\substack{C \in S \\ C \neq V}} \Pr(C \text{ votes 'yes'}) \prod_{C \in N \setminus S} (1 - \Pr(C \text{ votes 'yes'})) \quad (6)$$

The sum in (6) is taken over all those partitions of the Executive Board (which is denoted by N) into coalitions voting "yes" (the subset S) and "no" (the complement $N\setminus S$ since abstention from voting is not allowed) for which the constituency V makes a swing. For more details on derivation of this formula, see Aleskerov, Kalyagin &Pogorelskiy (2008).

Now for each country i of the constituency V the total power index is a product of α and κ, i.e.,

$$\pi(i, V) = \alpha(i)\kappa(V) \quad (7)$$

3.2 The assumptions regarding the governance of the Executive Board

- As a rule of decision making at the level of the Executive Board we separately consider all the three types of majority (namely, simple, qualified 70%, and 85% of the total votes).
- For decision making process at the constituency level the simple majority rule is used.
- All members have profiles of preferences to coalesce with other countries of their constituency, which are used as a basis for determining a country's power index within the constituency.
- A preference profile is a set of intensities of connection – these are interpreted as country *i*'s wish to form a coalition with country j – defined by bilateral trade with the country j in the following way

$$p_{ij} = \frac{X_{ij}/TX_i}{\sum_{\substack{k \in V \\ k \neq i}} X_{ik}/TX_i} = \frac{X_{ij}}{\sum_{\substack{k \in V \\ k \neq i}} X_{ik}} \quad (8)$$

Here $i, j \in V$ (both countries belong to the same constituency), X_{ij} is the absolute value of total exports from country i to j; TX_i is the total exports from country i to the world.

Note that (8) expresses the intensity of connection arising from bilateral trade as the share of exports of country i to country j in the total exports of country i to all countries of the respective constituency. The assumption is that a higher value of p_{ij} define a more important partner for country i.

The data on trade we used are of 2005, which is one of the most stable years of the period without major structural changes in the world economy (2000-2008). The data on exports are from the official IMF database "Direction of Trade Statistics", accessible at http://www.imfstatistics.org/dot/.

4 Preference-based voting power distribution within the IMF

The power analysis of the Executive Board was done using the data of May 2009. For every country we obtained three indices, reflecting its power in the Board under the majority rules considered.

4.1 Main results

The calculated power indices for all countries are given in the Appendix A.

As could be expected (recall the example from the introduction), the increase of the quota from the simple majority to the qualified majority of 85% lessens the absolute power of all constituencies.

It is worth noting that although the US have the maximal absolute power under a simple majority rule, if we take the majority of 70%, then the Belgian constituency has the greatest power and under the majority of 85% the Dutch constituency takes the lead.

We discuss two main reasons for this fact.

First, a constituency acquires most of its power because of its votes (as the quota increases, the smaller players significantly gain in the number of swings).

Second, due to a relatively small value of the respective probability to vote "yes" (defined by (5)) for both the Belgian and the Dutch constituencies, the winning coalitions organized by other members become less likely, which in turn decreases the power of other constituencies.

For every constituency we also calculated the Penrose and Banzhaf power indices, which were then compared to the κ indices and their normalized versions, respectively. The results of comparison are given in the tables below.

Table 4.1 – The case of simple majority

Constituency	Number of votes	Prob. to vote "yes" [7]	Penrose power index x10^6	κ power index x10^6	Banzhaf power index, %	Normalized κ power index, %
US	371,743	0.5000	635,879.99	154,325.85	20.9032	9.7871
Japan	133,378	0.5000	174,545.29	92,837.20	5.7378	5.8876
Germany	130,332	0.5000	170,598.51	90,400.95	5.6081	5.7331
France	107,635	0.5000	140,702.72	76,033.70	4.6253	4.8219
UK	107,635	0.5000	140,702.72	76,033.70	4.6253	4.8219
Belgian_C	113,969	0.0604	149,018.76	104,501.35	4.8987	6.6273
Dutch_C	105,937	0.0466	138,461.59	96,513.07	4.5516	6.1207
Mexican_Spanish_C	98,659	0.0756	128,894.33	87,939.65	4.2371	5.5770
Italian_C	90,968	0.5000	118,780.61	62,874.17	3.9047	3.9874
China	81,151	0.5000	105,904.10	57,320.37	3.4814	3.6352
Canadian_C	80,636	0.5000	105,167.39	56,908.80	3.4572	3.6091
Malaysian_C	78,068	0.0467	101,885.80	66,528.00	3.3493	4.2191

[7] For the constituencies of one country there are no reasons to assume any particular bias in voting. Therefore this probability is set to 0.5 in line with the general approach to *a priori* voting power analysis.

Australian_C	76,311	0.0585	99,575.04	64,308.54	3.2733	4.0783
Swedish_C	76,276	0.0859	99,527.84	63,645.40	3.2718	4.0363
Egyptian_C	70,852	0.0427	92,430.11	60,117.95	3.0384	3.8126
Saudi Arabia	70,105	0.5000	91,477.87	48,906.58	3.0071	3.1016
South_African_C	66,763	0.0313	87,041.38	56,374.32	2.8613	3.5752
Swiss_C	61,827	0.5000	80,591.20	43,110.24	2.6493	2.7340
Russia	59,704	0.5000	77,808.38	41,948.51	2.5578	2.6603
Iranian_C	53,662	0.0819	69,924.83	43,304.86	2.2986	2.7463
Brazilian_C	53,634	0.5000	69,892.41	38,122.50	2.2976	2.4177
Indian_C	52,112	0.5000	67,900.66	36,839.46	2.2321	2.3363
Argentinian_C	43,395	0.1150	56,480.41	34,429.77	1.8567	2.1835
Central_African_C	29,855	0.0241	38,830.76	23,509.38	1.2765	1.4909

Table 4.1 shows that for a simple majority case the values of Penrose index are always greater than those of κ power index. This is due to the fact that the Penrose power index assumes all coalitions equiprobable, which is equivalent to requiring each member's probability to vote "yes" or "no" be the same and equal 0.5. In this case every swing for a member adds $\frac{1}{2^{n-1}}$ to the total value of the index. For the κ power index each swing for a member adds the product of probabilities of those voting "yes" and those voting "no" to the total value. As these probabilities are generally different, their product might be different from $\frac{1}{2^{n-1}}$, depending on the division of the members in the ballot outcome. The calculations reveal that on average each swing "invests" less in the total value of the κ power index than in the Penrose one, affecting the resulting value which is also less. When normalizing it results in an increase in power share of the constituencies with the largest absolute power effectively decreasing the power shares of the remaining members. Thus, the Banzhaf index (i.e., the normalized Penrose index) is less than the κ power index for most constituencies.

Table 4. 2 - The case of qualified majority of 70%

Constituency	Number of votes	Prob. to vote "yes"	Penrose power index x10^6	κ power index x10^6	Banzhaf power index, %	Normalized κ power index, %
US	371,743	0.5000	103,230.95	93.27	11.0702	2.9192
Japan	133,378	0.5000	59,077.02	89.04	6.3353	2.7867
Germany	130,332	0.5000	57,801.49	88.47	6.1985	2.7690
France	107,635	0.5000	48,218.01	84.16	5.1708	2.6339
UK	107,635	0.5000	48,218.01	84.16	5.1708	2.6339
Belgian_C	113,969	0.0604	50,936.46	403.07	5.4623	12.6150
Dutch_C	105,937	0.0466	47,495.60	377.75	5.0933	11.8227
Mexico_Spanish_C	98,659	0.0756	44,329.40	290.61	4.7538	9.0953
Italian_C	90,968	0.5000	40,986.30	79.14	4.3953	2.4768
China	81,151	0.5000	36,663.53	74.91	3.9317	2.3445
Canadian_C	80,636	0.5000	36,432.98	74.70	3.9070	2.3381
Malaysian_C	78,068	0.0467	35,267.83	209.12	3.7820	6.5449
Australian_C	76,311	0.0585	34,498.93	194.28	3.6996	6.0806
Swedish_C	76,276	0.0859	34,479.38	176.21	3.6975	5.5149
Egyptian_C	70,852	0.0427	32,049.66	172.93	3.4369	5.4124
Saudi Arabia	70,105	0.5000	31,734.47	68.11	3.4031	2.1318
South_African_C	66,763	0.0313	30,275.58	166.38	3.2467	5.2074
Swiss_C	61,827	0.5000	28,033.97	62.71	3.0063	1.9627
Russia	59,704	0.5000	27,067.90	61.16	2.9027	1.9143

Iranian_C	53,662	0.0819	24,361.61	109.76	2.6125	3.4353
Brazilian_C	53,634	0.5000	24,349.93	56.29	2.6112	1.7616
Indian_C	52,112	0.5000	23,671.39	55.42	2.5385	1.7344
Argentinian_C	43,395	0.1150	19,736.05	73.27	2.1164	2.2932
Central_African_C	29,855	0.0241	13,592.00	50.20	1.4576	1.5711

Table 4.2 demonstrates that the abovementioned relation between the Penrose and κ indices also holds for the quota set at 70%; the Penrose index significantly decreases for all countries while the κ index simply plunges. In effect this means an increasing role of preferences for a higher quota. With regards to the normalized versions of indices, as preference-based absolute power tends to equalize, more countries express deviations in either direction from the benchmark case of the Banzhaf index.

Table 4. 3 – The 85% majority case

Constituency	Number of votes	Prob. to vote "yes"	Penrose power index x10^6	κ power index x10^6	Banzhaf power index, %	Normalized κ power index, %
US	371,743	0.5000	1313.09	0.0014	6.3671	1.5158
Japan	133,378	0.5000	1175.52	0.0014	5.7001	1.5103
Germany	130,332	0.5000	1165.99	0.0014	5.6538	1.5098
France	107,635	0.5000	1060.13	0.0014	5.1405	1.4918
UK	107,635	0.5000	1060.13	0.0014	5.1405	1.4918
Belgian_C	113,969	0.0604	1093.27	0.0105	5.3012	11.4302
Dutch_C	105,937	0.0466	1052.26	0.0124	5.1024	13.4564
Mexican_Spanish_C	98,659	0.0756	1008.63	0.0076	4.8908	8.3203
Italian_C	90,968	0.5000	952.12	0.0013	4.6168	1.4668
China	81,151	0.5000	877.50	0.0013	4.2550	1.4340
Canadian_C	80,636	0.5000	871.54	0.0013	4.2261	1.4327
Malaysian_C	78,068	0.0467	853.42	0.0084	4.1382	9.1844
Australian_C	76,311	0.0585	840.78	0.0073	4.0769	7.9141
Swedish_C	76,276	0.0859	840.54	0.0057	4.0758	6.1976
Egyptian_C	70,852	0.0427	792.38	0.0076	3.8422	8.2836
Saudi Arabia	70,105	0.5000	785.95	0.0013	3.8110	1.3999
South_African_C	66,763	0.0313	750.42	0.0074	3.6388	8.0102
Swiss_C	61,827	0.5000	705.36	0.0012	3.4203	1.3228
Russia	59,704	0.5000	685.10	0.0012	3.3220	1.2989
Iranian_C	53,662	0.0819	625.49	0.0039	3.0330	4.2790
Brazilian_C	53,634	0.5000	625.25	0.0011	3.0318	1.2328
Indian_C	52,112	0.5000	610.23	0.0011	2.9590	1.2173
Argentinian_C	43,395	0.1150	516.77	0.0025	2.5058	2.7320
Central_African_C	29,855	0.0241	361.08	0.0017	1.7509	1.8673

Table 4.3 shows the further decrease of the absolute voting power as the quota increases up to 85%. In effect this is the case where individual countries are not at all powerful on their own; it may indicate a need for a more general concept of power that could deal with such situations.

Turning to the analysis of the α power indices at the constituency level, note that taking into account countries' preferences almost always (except for the two extreme cases: a country with zero power and a dictator country, see below) decreases the value of this index compared to the analogous Penrose index. Indeed, it follows from (4) that for the α index to reach the value of

the Penrose index, we need to have maximum possible preferences values (equal 1) for all countries, which is not the case in reality.

From all constituencies we may pick out those having a "dictator"[8], i.e., a country with the number of votes exceeding 50% of all votes of a constituency[9]. For such countries both the α and Penrose indices equals 1.

For all other countries in such constituencies the α power index, as well as the Penrose one equals 0 as there have no swings (in total there are 41 countries with zero voting power, mostly developing ones (see appendix A)).

Besides, the absolute power indices may be 0 for a country even when the constituency does not have a "dictator", but the number of votes of this country does not allow it to have even a single swing. An example can be found in the Sweden constituency, where the power index of Estonia is zero.

In all other cases the values of the α power index are non-zero, but depending on the constituency structure may be very close to zero for some countries. Table 4.4 shows an example for the Belgian constituency.

Table 4. 4 – Voting power in the Belgian constituency

Members	Number of votes	α power index x10^4
Austria	18,973	102.54
Belarus	4,114	36.69
Belgium	46,302	1014.40
Czech Republic	8,443	94.34
Hungary	10,634	90.55
Kazakhstan	3,907	28.98
Luxembourg	3,041	45.11
Slovak Republic	3,825	33.38
Slovenia	2,567	8.44
Turkey	12,163	123.14

Table 4.4 states that Belgium have the maximal power (about 0.1), while the power of all other countries does not exceed 2×10^{-2}. Such small value of power is mostly determined by the fact that Belgium has 40.63% of the total votes of its constituency. But there is also another factor: the preference matrix of the constituency members[10] (Table 4.5), which could explain the difference in power between the countries with about the same number of votes. For instance, the possible number of swings for Belarus, Kazakhstan, Luxembourg and Slovak Republic is the same: 16; for the case of Belgium this value is 464. Hence, the difference in power between the

[8] These are, namely, Italian, Canadian, Swiss, Brazilian and Indian constituencies.

[9] Note that the analysis is performed under the assumption that all constituencies use the simple majority rule. The power distribution can be qualitatively different under some other decision making rule. However, our approach seems reasonable at least because of the statement from Articles of Agreement, Article XII, Section 3(f) about the re-elections of an Executive Director, performed by a simple majority voting.

[10] Every cell in the preference matrix corresponds to the preference measure of a country from the respective row to coalesce the one from the specified column.

countries with the same number of swings is determined just by their preferences, mainly those of coalescing with Belgium since it is pivotal in almost all winning coalitions.

Thus, for example, the power of Luxembourg is about 1.56 times greater than that of Kazakhstan, and Luxembourg's preferences to coalesce with Belgium are 0.6457, while those of Kazakhstan are just 0.0223. Note that Belgium's preference to coalesce is also greater for Luxembourg than for Kazakhstan.

Another example is the Czech Republic and Hungary. Hungary has a greater number of votes than the Czech Republic, but both have the same number of possible swings. The countries' preference to coalesce with Belgium is approximately the same, too, but Belgium prefers the Czech Republic to Hungary which is reflected in a greater value of the α power index for the Czech Republic.

These results confirm that under a greater majority rule voting power depends on voters' preferences to coalesce, which effect may outweigh the factor of the number of votes.

Table 4. 5 – The preferences of members of the Netherlands constituency

	Austria	Belarus	Belgium	Czech Republic	Hungary	Kazakhstan	Luxembourg	Slovak Republic	Slovenia	Turkey
Austria	-	0.0076	0.1313	0.2352	0.2669	0.0111	0.0147	0.1328	0.1377	0.0627
Belarus	0.0296	-	0.1386	0.0982	0.2370	0.3232	0.0087	0.0896	0.0087	0.0663
Belgium	0.1761	0.0063	-	0.1170	0.0882	0.0055	0.3668	0.0370	0.0246	0.1785
Czech Republic	0.2597	0.0059	0.1272	-	0.1265	0.0056	0.0068	0.4098	0.0270	0.0315
Hungary	0.3369	0.0091	0.1283	0.1830	-	0.0097	0.0034	0.1711	0.0701	0.0885
Kazakhstan	0.4179	0.0206	0.0223	0.1087	0.0434	-	0.0000	0.0063	0.0120	0.3689
Luxembourg	0.1568	0.0010	0.6457	0.0545	0.0299	0.0032	-	0.0179	0.0215	0.0695
Slovak Republic	0.2266	0.0040	0.0629	0.4507	0.1881	0.0031	0.0038	-	0.0302	0.0306
Slovenia	0.4646	0.0108	0.0665	0.1383	0.1381	0.0140	0.0184	0.0895	-	0.0598
Turkey	0.1824	0.0138	0.3577	0.0803	0.1049	0.1273	0.0077	0.0340	0.0920	-

5 Conclusions

For the Fund members we have calculated their preferences to coalesce based on their structure of exports. The preference-based power indices were calculated for the three cases of a quota value for a decision to be taken.

The analysis showed a remarkable difference between the values of the proposed indices and the classical Penrose and Banzhaf ones. We argue that under a greater majority voting rule the countries' preferences to coalesce become an important factor that in effect defines their power.

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Appendix A
Power indices of the IMF members

Table A. 1 -Power indices of the IMF members

Constituency*	Constituency members**	The α power index x10^6	The π power index x10^6 (simple majority)	The π power index x10^6 (qualified majority of 70%)	The π power index x10^6 (qualified majority of 85%)
US	US	1,000,000.0000	154,325.8466	93.2723	0.0014
Japan	Japan	1,000,000.0000	92,837.1990	89.0394	0.0014
Germany	Germany	1,000,000.0000	90,400.9478	88.4742	0.0014
France	France	1,000,000.0000	76,033.7027	84.1574	0.0014
UK	UK	1,000,000.0000	76,033.7027	84.1574	0.0014
Belgian_C					
	Austria	10,253.8763	1,071.5439	4.1330	0.0001
	Belarus	3,669.0786	383.4237	1.4789	0.0000
	Belgium	101,439.5745	10,600.5725	40.8869	0.0011
	Czech Republic	9,433.5831	985.8222	3.8024	0.0001
	Hungary	9,054.6984	946.2282	3.6496	0.0001
	Kazakhstan	2,898.1239	302.8579	1.1681	0.0000
	Luxembourg	4,510.9948	471.4050	1.8182	0.0000
	Slovak Republic	3,337.7145	348.7957	1.3453	0.0000
	Slovenia	843.5645	88.1536	0.3400	0.0000
	Turkey	12,313.5789	1,286.7856	4.9632	0.0001
Dutch_C					
	Armenia	250.7740	24.2030	0.0947	0.0000
	Bosnia and Herzegovina	94.2106	9.0926	0.0356	0.0000
	Bulgaria	129.7627	12.5238	0.0490	0.0000
	Croatia	95.6604	9.2325	0.0361	0.0000
	Cyprus	247.1427	23.8525	0.0934	0.0000
	Georgia	104.4286	10.0787	0.0394	0.0000
	Israel	302.4050	29.1860	0.1142	0.0000
	Macedonia, former Yugoslav Republic of	40.8529	3.9428	0.0154	0.0000
	Moldova	82.5975	7.9717	0.0312	0.0000
	Montenegro, Republic of	24.4141	2.3563	0.0092	0.0000
	Netherlands	83,244.7270	8,034.2044	31.4457	0.0010
	Romania	184.7014	17.8261	0.0698	0.0000
	Ukraine	140.5851	13.5683	0.0531	0.0000
Mexican_Spanish_C					
	Costa Rica	0.0000	0.0000	0.0000	0.0000
	El Salvador	0.0000	0.0000	0.0000	0.0000
	Guatemala	0.0000	0.0000	0.0000	0.0000
	Honduras	0.0000	0.0000	0.0000	0.0000
	Mexico	76,784.2344	6,752.3788	22.3141	0.0006
	Nicaragua	0.0000	0.0000	0.0000	0.0000
	Spain	80,752.6030	7,101.3557	23.4673	0.0006
	Venezuela	76,456.4212	6,723.5510	22.2188	0.0006
Italian_C					
	Albania	0.0000	0.0000	0.0000	0.0000
	Greece	0.0000	0.0000	0.0000	0.0000
	Italy	1,000,000.0000	62,874.1723	79.1377	0.0013

	Malta	0.0000	0.0000	0.0000	0.0000
	Portugal	0.0000	0.0000	0.0000	0.0000
	San Marino	0.0000	0.0000	0.0000	0.0000
	Timor-Leste	0.0000	0.0000	0.0000	0.0000
China	China	1,000,000.0000	57,320.3695	74.9101	0.0013
Canadian_C					
	Antigua and Barbuda	0.0000	0.0000	0.0000	0.0000
	Bahamas, The	0.0000	0.0000	0.0000	0.0000
	Barbados	0.0000	0.0000	0.0000	0.0000
	Belize	0.0000	0.0000	0.0000	0.0000
	Canada	1,000,000.0000	56,908.8013	74.7046	0.0013
	Dominica	0.0000	0.0000	0.0000	0.0000
	Grenada	0.0000	0.0000	0.0000	0.0000
	Ireland	0.0000	0.0000	0.0000	0.0000
	Jamaica	0.0000	0.0000	0.0000	0.0000
	St. Kitts and Nevis	0.0000	0.0000	0.0000	0.0000
	St. Lucia	0.0000	0.0000	0.0000	0.0000
	St. Vincent and the Grenadines	0.0000	0.0000	0.0000	0.0000
Malaysian_C					
	Brunei Darussalam	5,711.0668	379.9459	1.1943	0.0000
	Cambodia	2,189.3046	145.6501	0.4578	0.0000
	Fiji	1,905.5242	126.7707	0.3985	0.0000
	Indonesia	51,303.6250	3,413.1278	10.7286	0.0004
	Lao People's Democratic Republic	1,876.4757	124.8382	0.3924	0.0000
	Malaysia	32,341.4386	2,151.6114	6.7632	0.0003
	Myanmar	6,501.8562	432.5555	1.3597	0.0001
	Nepal	2,037.0726	135.5224	0.4260	0.0000
	Philippines	18,136.1186	1,206.5598	3.7926	0.0002
	Singapore	18,160.6136	1,208.1894	3.7977	0.0002
	Thailand	23,604.9521	1,570.3904	4.9363	0.0002
	Tonga	677.9553	45.1030	0.1418	0.0000
	Vietnam	7,757.3097	516.0783	1.6222	0.0001
Australian_C					
	Australia	45,106.2537	2,900.7175	8.7634	0.0003
	Kiribati	0.0000	0.0000	0.0000	0.0000
	Korea	44,440.3497	2,857.8942	8.6340	0.0003
	Marshall Islands	0.0000	0.0000	0.0000	0.0000
	Micronesia, Federated States of	0.0000	0.0000	0.0000	0.0000
	Mongolia	0.0000	0.0000	0.0000	0.0000
	New Zealand	45,286.5021	2,912.3090	8.7984	0.0003
	Palau	0.0000	0.0000	0.0000	0.0000
	Papua New Guinea	0.0000	0.0000	0.0000	0.0000
	Samoa	0.0000	0.0000	0.0000	0.0000
	Seychelles	0.0000	0.0000	0.0000	0.0000
	Solomon Islands	0.0000	0.0000	0.0000	0.0000
	Vanuatu	0.0000	0.0000	0.0000	0.0000
Swedish_C					

	Denmark	47,932.3938	3,050.6762	8.4461	0.0003
	Estonia	0.0000	0.0000	0.0000	0.0000
	Finland	33,691.0269	2,144.2788	5.9367	0.0002
	Iceland	3,969.5162	252.6414	0.6995	0.0000
	Latvia	4,514.3517	287.3177	0.7955	0.0000
	Lithuania	4,275.3078	272.1037	0.7533	0.0000
	Norway	47,158.7266	3,001.4358	8.3098	0.0003
	Sweden	109,376.7002	6,961.3234	19.2731	0.0006
Egyptian_C					
	Bahrain	4,051.0247	243.5393	0.7006	0.0000
	Egypt	23,756.5703	1,428.1964	4.1083	0.0002
	Iraq	31,439.1743	1,890.0588	5.4369	0.0002
	Jordan	4,927.1498	296.2102	0.8521	0.0000
	Kuwait	38,400.5770	2,308.5641	6.6407	0.0003
	Lebanon	5,767.2359	346.7144	0.9973	0.0000
	Libyan Arab Jamahiriya	29,902.9735	1,797.7055	5.1712	0.0002
	Maldives	960.8685	57.7654	0.1662	0.0000
	Oman	5,750.5875	345.7135	0.9945	0.0000
	Qatar	7,574.2850	455.3505	1.3098	0.0001
	Syrian Arab Republic	8,615.4404	517.9426	1.4899	0.0001
	United Arab Emirates	18,485.0175	1,111.2814	3.1967	0.0001
	Yemen, Republic of	6,844.9564	411.5048	1.1837	0.0001
Saudi Arabia	Saudi Arabia	1,000,000.0000	48,906.5831	68.1138	0.0013
South_African_C					
	Angola	7,716.7455	435.0263	1.2839	0.0001
	Botswana	2,107.4205	118.8044	0.3506	0.0000
	Burundi	2,356.0137	132.8187	0.3920	0.0000
	Eritrea	922.6353	52.0129	0.1535	0.0000
	Ethiopia	3,694.2747	208.2622	0.6147	0.0000
	Gambia, The	1,327.9436	74.8619	0.2209	0.0000
	Kenya	7,079.3763	399.0951	1.1779	0.0001
	Lesotho	1,391.0198	78.4178	0.2314	0.0000
	Liberia	3,559.0374	200.6383	0.5922	0.0000
	Malawi	2,247.8178	126.7192	0.3740	0.0000
	Mozambique	3,310.4115	186.6222	0.5508	0.0000
	Namibia	3,883.2073	218.9132	0.6461	0.0000
	Nigeria	27,578.0971	1,554.6966	4.5886	0.0002
	Sierra Leone	3,034.2075	171.0514	0.5048	0.0000
	South Africa	29,477.0545	1,661.7490	4.9045	0.0002
	Sudan	4,506.9489	254.0762	0.7499	0.0000
	Swaziland	1,800.0183	101.4748	0.2995	0.0000
	Tanzania	5,442.3115	306.8066	0.9055	0.0000
	Uganda	4,817.8471	271.6029	0.8016	0.0000
	Zambia	14,366.7762	809.9173	2.3904	0.0001
Swiss_C					
	Azerbaijan	0.0000	0.0000	0.0000	0.0000
	Kyrgyz Republic	0.0000	0.0000	0.0000	0.0000
	Poland	0.0000	0.0000	0.0000	0.0000
	Serbia, Republic of	0.0000	0.0000	0.0000	0.0000
	Switzerland	1,000,000.0000	43,110.2391	62.7110	0.0012
	Tajikistan	0.0000	0.0000	0.0000	0.0000
	Turkmenistan	0.0000	0.0000	0.0000	0.0000
	Uzbekistan	0.0000	0.0000	0.0000	0.0000

Russia	Russia	1,000,000.0000	41,948.5130	61.1633	0.0012
Iranian_C					
	Afghanistan, Islamic Republic of	12,659.7062	548.2268	1.3896	0.0000
	Algeria	68,561.5496	2,969.0481	7.5256	0.0003
	Ghana	20,702.3062	896.5104	2.2724	0.0001
	Iran, Islamic Republic of	92,333.9401	3,998.5081	10.1349	0.0004
	Morocco	22,818.9595	988.1718	2.5047	0.0001
	Pakistan	60,350.2394	2,613.4585	6.6243	0.0002
	Tunisia	9,895.4259	428.5200	1.0862	0.0000
Brazilian_C					
	Brazil	1,000,000.0000	38,122.5009	56.2866	0.0011
	Colombia	0.0000	0.0000	0.0000	0.0000
	Dominican Republic	0.0000	0.0000	0.0000	0.0000
	Ecuador	0.0000	0.0000	0.0000	0.0000
	Guyana	0.0000	0.0000	0.0000	0.0000
	Haiti	0.0000	0.0000	0.0000	0.0000
	Panama	0.0000	0.0000	0.0000	0.0000
	Suriname	0.0000	0.0000	0.0000	0.0000
	Trinidad and Tobago	0.0000	0.0000	0.0000	0.0000
Indian_C					
	Bangladesh	0.0000	0.0000	0.0000	0.0000
	Bhutan	0.0000	0.0000	0.0000	0.0000
	India	1,000,000.0000	36,839.4614	55.4164	0.0011
	Sri Lanka	0.0000	0.0000	0.0000	0.0000
Argentinian_C					
	Argentina	187,500.0000	6,455.5813	13.7383	0.0005
	Bolivia	21,962.3490	756.1586	1.6092	0.0001
	Chile	16,576.1027	570.7114	1.2145	0.0000
	Paraguay	11,407.6931	392.7642	0.8358	0.0000
	Peru	8,771.7033	302.0077	0.6427	0.0000
	Uruguay	21,791.1332	750.2636	1.5967	0.0001
Central_African_C					
	Benin	3,450.7180	81.1242	0.1732	0.0000
	Burkina Faso	3,382.8072	79.5277	0.1698	0.0000
	Cameroon	8,524.9764	200.4169	0.4279	0.0000
	Cape Verde	1,382.9680	32.5127	0.0694	0.0000
	Central African Republic	3,595.3763	84.5251	0.1805	0.0000
	Chad	3,209.4731	75.4527	0.1611	0.0000
	Comoros	1,353.2284	31.8136	0.0679	0.0000
	Congo, Democratic Republic of	28,051.7933	659.4803	1.4081	0.0000
	Congo, Republic of	4,431.4285	104.1801	0.2224	0.0000
	Cote d'Ivoire	14,035.0573	329.9555	0.7045	0.0000
	Djibouti	1,624.1224	38.1821	0.0815	0.0000
	Equatorial Guinea	2,315.5018	54.4360	0.1162	0.0000
	Gabon	7,558.8233	177.7032	0.3794	0.0000
	Guinea	5,334.1088	125.4016	0.2678	0.0000
	Guinea-Bissau	1,557.1733	36.6082	0.0782	0.0000
	Madagascar	5,890.5185	138.4824	0.2957	0.0000

	Mali	4,709.6796	110.7216	0.2364	0.0000
	Mauritius	5,074.2163	119.2917	0.2547	0.0000
	Niger	3,623.0021	85.1745	0.1819	0.0000
	Rwanda	4,748.0258	111.6231	0.2383	0.0000
	Sao Tome and Principe	1,305.9450	30.7020	0.0656	0.0000
	Senegal	7,512.2146	176.6075	0.3771	0.0000
	Togo	3,906.7413	91.8451	0.1961	0.0000

Notes:

* The titles of the constituencies are formed from the names of the countries, providing the Executive Directors for these constituencies. The five countries with the largest quotas (the USA, Japan, Germany, France, the UK) appoint their Executive Directors. Three countries (China, Saudi Arabia, Russia), having the number of votes sufficient to elect the Executive Director without the need for creating a constituency, in effect, also appoints their Directors. Therefore the α power indices for these countries as well as for those with the number of votes exceeding 50% of the total votes in a constituency, equal 1. For all the remaining countries in such constituencies their power indices equal 0.

** The countries, which did not participate in the elections due to suspension of the voting rights, or other reasons, are not listed in the table.